

**NETWORK BASED EVALUATION METHOD
FOR FINANCIAL ANALYSIS OF TOLL ROADS**

A Thesis

by

NEVENA VAJDIC

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2009

Major Subject: Civil Engineering

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Approved by:

Chair of Committee,	Ivan Damnjanovic
Committee Members,	Stuart D. Anderson
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ABSTRACT

Network Based Evaluation Method

For Financial Analysis of Toll Roads. (December 2009)

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Chair of Advisory Committee: Dr. Ivan Damnjanovic

The design, build, finance and operation of public infrastructure is becoming increasingly dependent on participation of the private sector. An imposing amount of investment involved in a public private partnership agreement places financial institutions in the role of major lenders. The complexity of these agreements creates a gap in the information flow between the public sector, the private sector and financial institutions as project participants. Additionally, the public sector decisions about the network improvement actions add to the complexity of these agreements. The objective of this research is to develop a method which will allow an assessment of the effect that network improvement actions have on the project's financial feasibility. Three common financial instruments were analyzed: bank loans, bonds and real options. Emphasis of the financial feasibility assessment was on the price of the revenue risk, as the most important risk in public private partnership agreements. Results have shown that network improvement actions can have significant impact on the price of the revenue risk. The magnitude of the impact depends on the type of instrument and the position of the road link in the network.

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CHAPTER I

INTRODUCTION

The public need for improved and expanded transportation networks constantly increases, thus placing higher pressure on the public sector for available funds needed for network improvements. In order to bridge the gap between existing and needed funds, the public sector is looking into alternative methods of financing projects. Although the idea and concept of Public-Private Partnerships (PPP) reaches back in history, in recent years the advanced version of this method has served as an alternative for project delivery.

BACKGROUND

This research was conducted for the Texas Department of Transportation (TxDOT) Research Project No. 0-5881: *Quantifying the Effects of Network Improvement Actions on the Value of New and Existing Toll Road Projects*, in the joint research work with the University of Texas in Austin. The purpose of this research was to develop a decision – support process for TxDOT. This process will serve as a tool for the evaluation of effects that improvement action on the existing network will have on toll roads risk-dependent measures. More importantly, this tool will allow an objective evaluation of which network improvements will add the most value to toll road projects.

This thesis follows the style of the *Journal of Construction Engineering and Management*.

The focus of TxDOT 0-5881 research project was on two research areas: modeling network enhancements and project risk management. These two research areas were divided into four major domains: identification, quantification, specification, and optimization. The project was further divided into 11 tasks. The overview of major domains along with the position of nine tasks in the project, as it was presented in the work plan section of the research project proposal, is presented in Fig. 1-1.

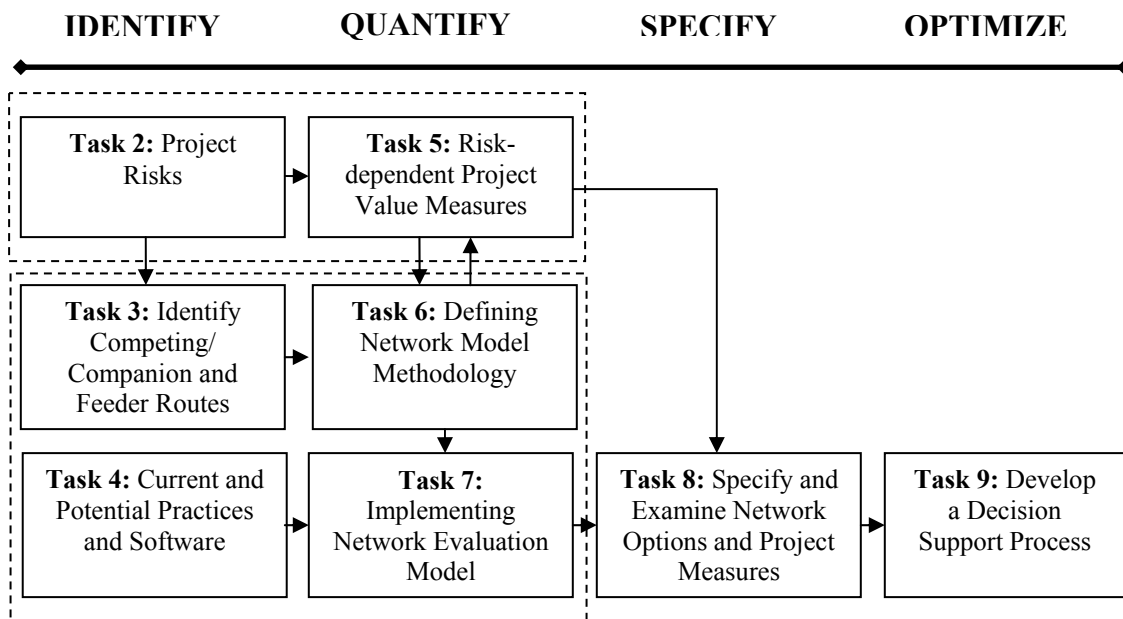


Fig. 1-1. Overview of project tasks

The project started with the literature review (Task 1, not included in the Fig. 1-1) followed with the identification of risks in toll roads. The third task developed the process for the identification of competing and feeder routes in the network. In the fourth task, the overview of TxDOT current practices and software was presented with the

discussion of potential practices. Completion of this task concluded the project area related to the identification.

The purpose of task five was to select and integrate risk-dependent measures used for the project valuation with decision making process. The sixth task was focused on the definition of network model which will help in accomplishment of research objective. In task seven, the implementation of network evaluation model was delivered with the emphasis on the simple case study.

Task eight was focused on the selection of specific risk dependent measures which have significant impact on the project value. Task nine provided the developed decision-support process. Case study was conducted for the Austin network in task ten. Task 11 is a final research report, currently under preparation in the time when this thesis is written.

The research for this thesis was focused on the project risk management area, while the modeling network enhancements area was researched by the University of Texas in Austin. Thus, this thesis represents the compilation of the research done for tasks two, five, eight and nine. However, in order to present the decision-support process developed in this research, the case study is presented for the simple network. The details of identification of competing and feeder links, current software and practices, as well as the integration of the network model with existing TxDOT tools are not included in this thesis. The context of the project risk management area for toll roads used in this research is presented in the following section, that is, the context of this thesis.

Toll Road Risk Management

Project finance is one type of PPP where private sector and lenders are considering a set of forecasted revenues for their return on the investment and debt-service assessment (Yescombe, 2002). However, when arrangements between the public sector, the private sector, and lenders are negotiated, revenues can only be forecasted and the uncertainty of the future outcome makes the revenue the main driver of risk assessment in project finance.

In order to reduce this risk, many strategies have been developed over the years for possible risk mitigation. One of the strategies is a non-competing clause in the agreement for toll roads that constraints the public sector from improving the existing road network within some distance from the road of interest (Ortiz and Buxbaum, 2008). Otherwise, the public sector needs to pay the shortfall in the revenue to the private investor. Other strategies can be used as well like real options, which can reduce the risk exposure for all project participants (Nevitt and Fabozzi, 1995). These real options provide flexibility to the public or private sector to prevent potential losses or increase the profits.

Infrastructure projects like toll roads with usage risk usually have debt to equity ratio of 80:20 (Yescombe, 2002). This means that the level of debt raised for the project will be at this ratio with the amount of equity invested. There are two main sources of project finance debt: commercial banks and bonds. Since banks are flexible when it comes to the renegotiating loan terms, in the case when the borrower experiences problems in debt servicing, these loans are more appropriate for the early phases of

project development like the construction phase and the start-up phase. For the project's operational phase, bonds are more suitable as the instrument for capital raising since the project has established a certain level of continuous operation and some trends in cash-inflows can be assessed at this phase.

In either case, the risk assessment on lender side is again focused on the forecasted project's revenue. The credit worthiness of the project will be estimated solely on the future, and uncertain, revenue stream. It is critical for both the borrower and the lender to price this risk properly. Over the years, many regulations were established with the intention to standardize main principles in risk assessment and risk pricing for different financial agreements between lenders and borrowers. Nevertheless, due to the complex and specific nature of the PPP agreements, some regulations deal with the project finance agreements in general, leaving the risk assessment and risk pricing to the judgment of experts, as is the case with regulations for the banking business (Gatti *et al*, 2007)

In this context, public agencies, private investors and financial institutions are putting a great effort into the understanding and development of PPP characteristics and financial models. This is being accomplished either through research efforts or through data gathering from established road concessions throughout the world in an attempt to learn from experience. As a part of this process, this research is dealing with understanding of the consequences of certain decisions. More specifically, the effect decisions made by the public sector regarding the improvements of the surrounding network will have on the financial feasibility of the toll road project under negotiation is

studied. Particularly, some of the public policies are part of the long term planning process and their implementation can affect the project's performance under the PPP scheme either positively or negatively, thus potentially adding to the value of the project or detracting from the value of the project.

PROBLEM STATEMENT

Many questions arise in the negotiation phase between project participants in toll road agreements. In order to ensure the successful communication between those participating in the negotiation phase, it is important to identify and quantify available information related to the project. This identification and quantification should also include information about changes in the network structure due to network improvement actions. Since network improvements are part of the long-term planning process in public agencies, they will be implemented some time in the future, and, consequently, change the distribution of traffic flows in the network. The question is: *how will these network improvements affect the revenue on the toll road and the project's financial feasibility?*

RESEARCH OBJECTIVES

This research considers three common financial instruments used in PPPs: real options, bonds and bank loans. The first provides flexibility for the public or the private sector to reduce and mitigate the revenue risk in the project's negotiation phase. Loans and bonds are widely used for debt raising and they are negotiated between private parties which operate the project and financial institutions. For each of these instruments, different models are used for the risk assessment and the risk pricing, which will be addressed in detail in this thesis.

The objective of this research is to develop a method that will create a connection between mathematical models used for risk pricing for these financial instruments and the network structure. This connection will allow assessment of the impact that changes in the network structure have on financial instruments. The purpose of the method will be to: 1) provide a solid basis for the objective assessment of the impact that decisions made by the public sector, regarding the network improvements, have on financial instruments used in toll road agreements; 2) provide a tool for quantification of the impact that these changes in the network structure have on the project's financial feasibility; and 3) build a common ground which ensures that the decisions made by the public sector are interpreted in a meaningful manner for project managers and decision makers, that is, the public sector, the private sector, and financial institutions.

For better understanding of the behavior of financial parameters with respect to different network improvements, sensitivity analysis of these parameters is performed

for different values of the links parameters, that is, different incremental increase or decrease of the links capacities in the network.

EXPECTED BENEFITS

The evaluation method correlates components of PPP agreements that are related to three project participants: the public sector, the private sector, and financial institutions. From the literature review, it has been noted that existing methods used for risk pricing of financial instruments in toll road agreements do not consider and quantify effects of public sector actions like decisions to add the capacity to the network. Thus, it is anticipated that this lack of assessment of impact of such decisions creates a gap in the information flow between project participants. Some instruments, such as non-competing clause priced as the real option, are used to address private sector concerns about possible future actions from public sector. However, this type of clause only relates the associated risk with the pre-specified level of revenue without objective assessment of the impact that considered network improvement will have on the revenue stream.

The evaluation method establishes a framework for quantification of network improvement effects on financial instruments used in toll road agreements. This method creates a common ground for project participants, thus creating a valuable tool useful in the negotiation phase. Managers and decision makers benefit from this method because it provides an objective basis for the quantification of the public sector actions.

Sensitivity analysis of financial instruments with the respect to different increases or decreases in links capacities helps in understanding the effect this may have on project's financial feasibility.

ORGANIZATION OF THE STUDY

The first chapter provides the background of this research with the problem statement, research objectives and expected benefits. Chapter II continues the thesis with the in-depth literature review with emphasis on the uncertainty in the traffic and the revenue realization, bank loans, bonds, and real options. Research methodology is explained in Chapter III. Chapter IV explains basic principles of the network assignment process along with the introduction of the mathematical model used for the forecast of future traffic flow and revenues on the toll road. A simple network example is introduced as a case study. Chapters V, VI and VII explain and define main principles in risk pricing models used for bank loans, bonds and real options, respectively. Each of these chapters has a sub-chapter dedicated to the sensitivity analysis of parameters with respect to different network topologies. In Chapter VIII, obtained results from sensitivity analysis are discussed. Chapter IX concludes this thesis with conclusions and recommendations for the future research.

CHAPTER II

LITERATURE REVIEW

The literature review was conducted for the purpose of identification of current practices in toll road agreements. Special emphasis was placed on the review of key risk parameters, associated uncertainties, financial instruments and risk mitigation strategies. Web-based search engines and library resources were primarily used for this literature review. Also, various research reports were reviewed like World Bank research reports since this institution is one of major participants in PPP agreements in undeveloped and developing countries. Once when the literature was gathered, relative information were organized in several groups: principles of project finance, current practices in transportation sector, traffic and revenue forecast, and financial instruments used in project finance.

PREVIOUS WORK

Project Finance

Public-private partnership is an arrangement between the public and the private sector for development or delivering of a public service or public infrastructure (Hardcastle and Boothroyd, 2003). Project finance is one type of PPP with non-recourse or limited recourse finance principle (Yescombe, 2002). Non-recourse finance is an agreement where investors do not provide any guarantees for the project finance debt;

limited recourse financing is any arrangement where investors provide only a limited guarantee. Nevitt and Fabozzi (1995) explain that the final goal in project financing is an arrangement which will be beneficial for the investor without affecting their credit status. This is achieved through the, above mentioned, non-recourse financing.

Yescombe (2002) defines three major project risks categories: commercial risks, macro-economic risks, and political risks. Commercial risks are project specific risks, macro-economic risks are external financial risks like inflation risks, and political risks are country specific risks. However, at the center of the task of risk assessment for project finance is the revenue risk. The revenue is the source for debt repayment and for creating returns to investors.

In the report prepared by the World Bank and Ministry of Construction, Japan (1999), 18 different countries are analyzed for the purpose of the review of recent toll roads experiences in those countries. It is noticed that in all countries, the participation of private sector is significantly present, even for those countries that had tradition of toll-free roads.

Transportation Sector

In the World Bank report (World Bank and Ministry of Construction, Japan, 1999) as one of key issues for the successful implementation of public-private partnerships agreements for toll roads is a strategic network planning. In countries where entities involved in the toll road program were established isolated from the general network expansion planning program, there was a problem with a coordination and

information exchange. Other issues which public transportation sector and government is facing, beside planning and institutional issues, are legal and regulatory issues, concession contracts and government supports.

A committee formed to prepare a report on current practice of travel forecast models for the Transportation Research Board (2007) reported that the models used by Metropolitan Planning Organizations are in use for over fifty years and cannot address all new policy concerns. Alexander, Estache, and Oliveri (2000) analyze different methodological problems that regulatory agencies and policy makers face in the transport sector. They analyze review of concessioner contracts and price regulations in order to establish the link between authorities' impact on the market risk associated with the transportation projects.

Edwards and Bowen (2003), in their analysis of the communication in PPPs, point out that the transfer of information between two parties is successful if the information is meaningful to both of them. Different perceptions of risks for different parties involved place a special emphasis on risk communication in PPP agreements.

Uncertainty in Traffic and Revenue Realization

Through history of PPP agreements, there were many examples of overestimated or underestimated traffic forecasts. Flyvbjerg et.al. (2005) report that half of the road projects from the sample of 210 projects had an error in the traffic forecast and actual traffic of 20 percent. For example, for the Dulles Greenway assumption was that the traffic demand will increase at 14 percent rate for the first six years (Garvin and Cheah,

2004). However, the original estimate of 34,000 vehicles per day showed that the forecast was too optimistic and the actual average traffic was 11,500 vehicles per day in the first six months (Fishbein and Babbar, 1996). For these reasons of the traffic forecast errors, the government of Chile, for example, provided three possible values for the traffic growth: 4, 4.5 and 5 percent, for the offering of concession agreement (Vassallo, 2006).

To model the uncertainty in the future revenue, Irwin (2003) and Brandao and Saraiva (2008) use properties of the stochastic process such as a geometric Brownian motion for the assessment of an option value in infrastructure projects. The type of option analyzed in this model is the minimum traffic guarantee and it was modeled as the European option. Chiara and Garvin (2007) use two different methods for evaluating minimum revenue guarantee: the multi-least square Monte Carlo method and the multi-exercised boundary method. This guarantee includes a minimum level of revenue that is assured to the investor. If the real revenue falls below the specified level, the public sector (the government) has an obligation to pay the difference. Chow and Regan (2009) use stochastic processes such as geometric Brownian motion to model a future travel demand as the key concept in a real options analysis for managerial flexibility in network investments.

Bank Loans

Although projects can be financed from different sources, bank loans are widely used for financing the infrastructure projects (Brealey *et al.*, 1996, Nevitt and Fabozzi,

1995). Yescombe (2002) reports the growth for project finance bank loan obligations grew from 1996 at \$42,830 millions in loans, to \$108,447 millions in 2001 (source Project Finance International). There are 20 major banks in the field of project finance with more than 70 percent participation in all project finance loans.

As the nature of banking business is not simple and includes different types of agreements, models that banks use for risks assessment are very complex. Different regulations were developed with the intention of standardizing those models. For example, new Basel II Accord (Basel Committee on Banking Supervision, 2004) is a regulatory document that regulates banks exposure to loan risk. For each loan the bank lends, it needs to provide an adequate amount of a capital to cover potential losses. The Basel Committee, in this document, recognizes the probability of default, unexpected losses, expected losses and losses given default as the key parameters for credit risk analysis. However, there is a lack of quantitative models for the assessment of capital requirements for project finance loans according to the new Basel II Accord requirements (Gatti *et al.*, 2007).

Bonds

One of the possible sources which the private sector can use to raise additional funds during the project's operational phase are bonds (Yescombe, 2002). Nevitt and Fabozzi (1995) report that the use of bonds as the potential source of debt funds for infrastructure projects has increased and it is forecasted that this trend will continue in

the future. For example, in 1996 the total of project finance bonds value was \$4,791 millions while in 2001 the total was \$25,003 millions (Yescombe, 2002).

There are three main quantitative methods for pricing of bonds used in the credit risk analysis: structural, reduced, and incomplete information approach (Giesecke, 2004). At the center of the credit risk assessment for bonds is the probability of default or probability that the failure of the bond issuer to fulfill the financial agreement with bond investors will occur.

There are several approaches used for credit risk models. Crouhy *et al.* (2000) in their study analyze and compare four credit risk models. Credit migration approach is based on the assessment of probability for moving from one credit grade to another over some time horizon. Structural approach is based on the assessment of the asset value and the probability to default on debt service. Actuarial approach assumes that the probability of default, as the only parameter of interest, follows the Poisson process. The last credit risk model is addressing the probability of default as function of macro-variables.

Options

An option represents the contract between two parties which grants the right to one party, but not the obligation to buy or sell an asset for a pre-specified price (Trigeorgis, 1998). This right can be exercised at before or on pre-specified date. If the option can be exercised before the end of the contract, it is called an American option; if it can be exercised only at the end of the contract, it is called a European.

The theory of the option pricing dates back to Merton (1973), who derived explicit formulas for pricing European call and put options, and the options with the boundary condition (down-and-out). Rubinstein and Reiner (1991) and Rich (1994) developed pricing formulas for four types of European boundary options: down-and-out, down-and-in, up-and-out and up-and-in. Kunimoto and Ikeda (1992) extended the problem by developing the valuation formula for European options with curved boundaries. Geman and Yor (1996) used a Laplace transform for a derivation of a pricing formula based on the fundamental properties of Brownian motion. All these methods are developed for pricing of options based on the price of an underlying asset, its volatility, and the exercised price as a function of the asset price.

SUMMARY

Through the literature review, the main principles of project finance are identified as the starting point of further analysis of toll roads agreements and their financial feasibility. Also, current practices in the transportation sector are identified with emphasis on communication between participants in these agreements. Methods and models used for the addressing of uncertainties in the traffic and the revenue realization are presented. The emphasis of the remaining literature review was on financial instruments used in toll road agreements: bank loans, bonds, and real options. After the literature review was conducted, the research framework and method were developed, which is presented in the following chapter.

CHAPTER III

RESEARCH METHODOLOGY

This research was conducted through several steps: a literature review, development of a research framework, development of a method, analysis of existing models for risk pricing and testing of the method on a simple case study. The literature review served as a basis for understanding main principles and methods related to the research problem. In the research framework, key elements in the toll road agreements were identified which provided an objective basis for the development of the method. The method is further expanded with the analysis of mathematical models used for risk pricing for financial instruments, or, in this case, for the revenue risk pricing. Sensitivity analysis served as a basis for a better understanding of the effects of different network improvements. Also, this sensitivity analysis was used to validate the method through comparison of expected and achieved results.

RESEARCH FRAMEWORK

For the development of the tool for TxDOT which will allow an objective assessment of which network improvements can add the value to the existing and new toll roads, the research framework was developed. This framework was primarily focused on three elements:

- Identification of project participants in toll road agreements;

- Identification of a role of all project participants;
- Defining the value of the project.

In order to better understand the connections among project participants and their interaction, project participants were first identified. Yescombe (2002) defines project participants for a typical toll road agreement. This definition includes seven participants: a government, a contracting authority, an operator, a contractor, an investor, lenders and road users. Without loss of generality, for this research it is assumed that the government and the contracting authority can be represented as one entity – the public sector, namely. Also, it is assumed that the operator, the contractor and the investor can be assimilated in one entity, namely the private sector. The overview of project participants in the context of this research is presented in Fig. 3-1.

The second element of the research framework was to identify the role of each project participant. Following Yescombe's (2002) explanation, connections among project participants are also presented in Fig. 3-1 (adapted from Yescombe, 2002). The public sector offers the concession agreement for the toll road. The private sector, when it accepts and signs the agreement, provides equity expecting a return on the investment. Lenders are investing a significant amount of money creating the project's debt. Road users, as the most important participant, are using a toll road, and through toll payments, they are creating the revenue.

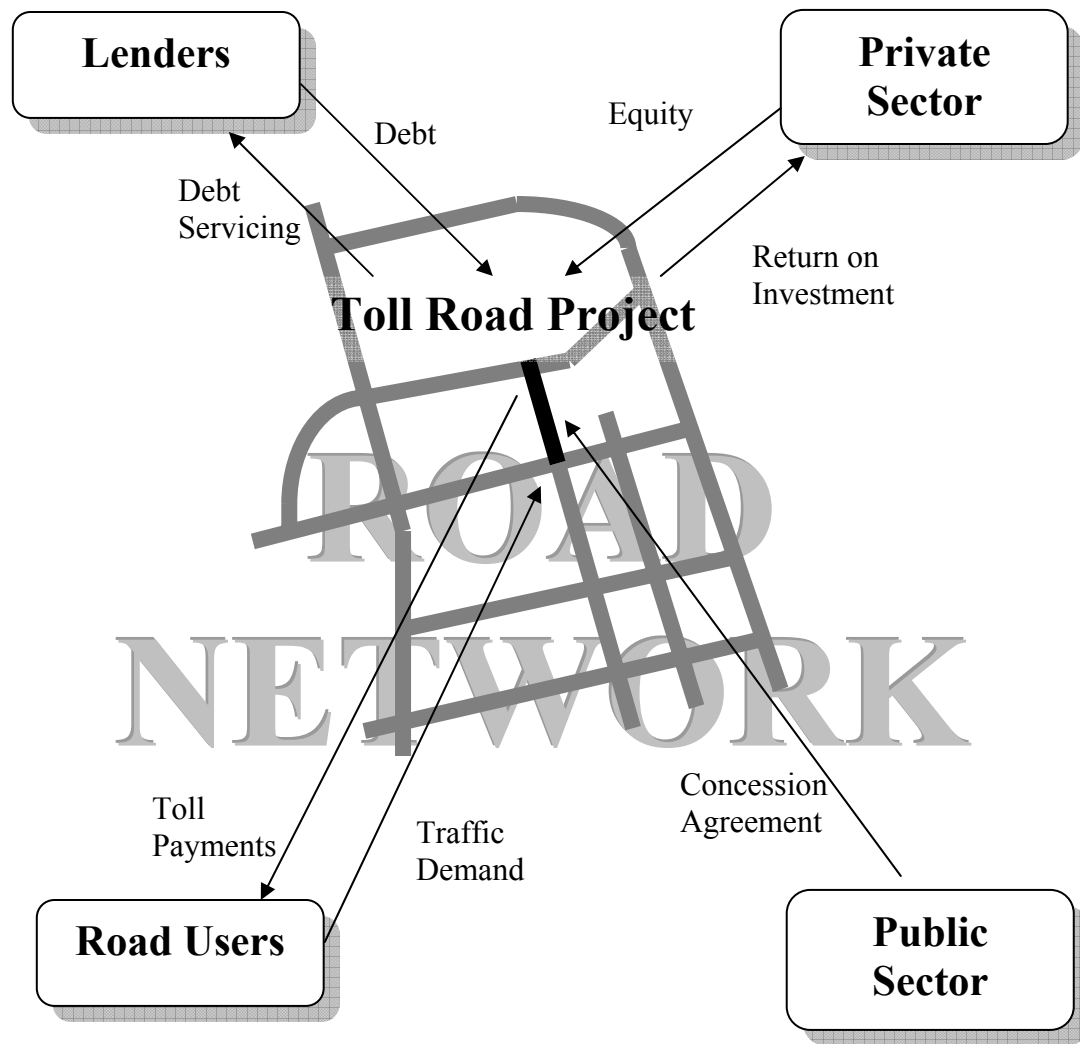


Fig. 3-1. Toll road project participants

However, interactions of project participants are much more complex. The public sector, before offering the concession agreement, goes through a project development phase. This development phase is part of the long-term planning process when decisions regarding the future network topology are made. When the project is in later stage of the

development phase, the concession agreement is offered. The most common structure of the concession agreement for toll roads is Build-Operate-Transfer (BOT) (Yescombe, 2002). This form of the agreement is also known as Design-Build-Finance-Operate (DBFO). Nevertheless, when the public sector is offering the concession to the private sector, usually it is asked to provide some form of guarantees or fiscal support for the project (Irwin, 2003). These guarantees between the public and the private sector serve as a tool for the risk mitigation. Thus, the interaction between the public sector and the private sector is mainly tied to the negotiation phase when the concession agreement is offered.

By the definition of the BOT scheme, the private sector provides the services of project design, construction, operation and finance. As mentioned earlier, the most common ratio between the debt and equity invested in the toll road is 80:20. Thus, beside the equity which it provides, the private sector is looking into available sources of funding which have the capacity to provide this significant amount of debt. This creates the connection between lenders and the private sector in toll road agreements. As discussed earlier, the main principle in toll road agreements is non-recourse financing. Non-recourse finance is an agreement where investors do not provide any guarantees for the project finance debt. Thus, lenders are relying on the estimates of project's future cash flows for their assessment of the risk.

The connection between road users and other project participants is not direct. Road users do not participate in the negotiation about the concession agreement nor do they participate in the financing of the project. However, they use the road and pay the

toll rate for each trip they make. The number of trips they made and the price of the toll rate will determine the amount of the revenue which is collected on that road. Thus, they are indirectly connected with other project participants as they are primary source of the revenue. As mentioned earlier, the revenue risk is the main risk in the toll road concessions. Hence, the revenue risk is tied to the number of trips that road users make or, in other words, to the traffic risk.

A third element of the research framework was to identify and specify the value of the project in toll road concessions. As mentioned, this research was focused on the development of the decision-support process that will allow the assessment of which network improvements add the value to the existing and planned toll roads. In this context, adding the value to the toll road project was considered as decreasing the price of risks. Explained in detail, the value of the toll road project is determined based on the expected cash inflows. The road itself is not a traded asset and the worthiness of the project is estimated solely on forecasted cash flows. Since these cash flows are forecasted and, hence, uncertain, the worthiness of the project will reflect this risk. The higher the risk, the project is worth less. The price of accepting and bearing the revenue risk will be high if the uncertainty in the future revenue is high. Thus, adding value to the project means decreasing this risk, or, in other words, decreasing the price of the revenue risk. Vice versa also holds, decreasing the value of the project means increasing the price of the revenue risk.

This research framework provided an insight into the key elements of toll road agreements. Main project participants were identified as well as their connections and

interactions. The direction of the research is placed in the context of the project's value and what it means to add or subtract the value of the toll road project. Based on this research framework, the evaluation method as the decision-supporting tool is specified in the following section.

EVALUATION METHOD

In order to develop the procedure for the evaluation method for financial analysis of toll road projects, several steps are proposed. The evaluation method identifies the network-based framework subject to changes initiated by the public sector and correlates these changes with the financial instruments used in PPP agreements (Fig. 3-2).

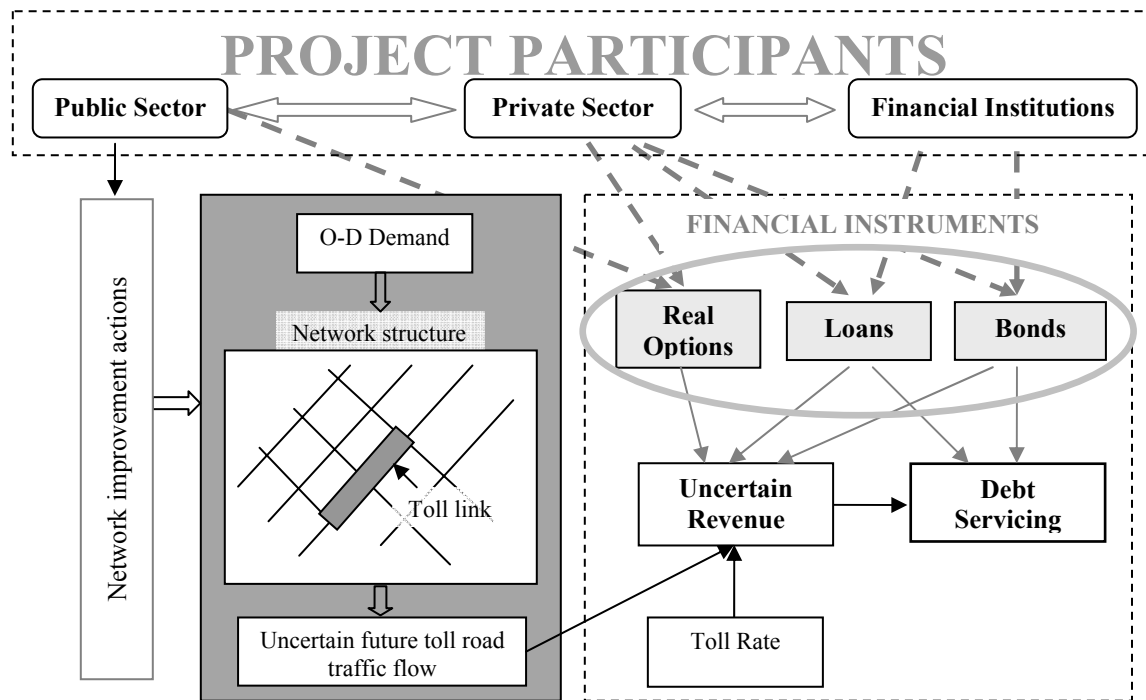


Fig. 3-2. Network-based evaluation method

The first step in the proposed methodology is to assign deterministic Origin-Destination (O-D) traffic demand to the existing network structure. This step is known as the traffic assignment process. Also, this step integrates the correlation of road users with other elements. For the given O-D demand, that is, the estimated number of trips between origin and destination, the number of trips on each link in the network can be determined through a process called traffic assignment. Network topology, links characteristics, and parameters determine the distribution of trips among network links. Obtained results from this step present the traffic flow on all links, including the toll link. Thus, solving the traffic assignment process, traffic flow on the toll link becomes known. Forecast of the future traffic on toll link is the next step. Here, a random process (geometric Brownian motion) is used for this purpose. The properties and formulation of geometric Brownian motion as well as mathematical formulation of traffic assignment process are explained in detail in Chapter IV.

The next step is to correlate the uncertain traffic flow with the revenue. The revenue is modeled as the function of the traffic flow and the toll rate. Since the traffic flow is uncertain and modeled as a random process, revenue becomes a random process also. This step introduces the model for the forecast of future uncertain revenue stream. Further, correlation between pricing models for considered financial instruments with this uncertain revenue is developed. As it will be explained in following chapters, pricing of real options is related only to the uncertain revenue forecast. However, bonds and bank loans will create a debt service for the project, which is expected to be repaid

from the project's cash flow over debt service life. Risk assessment and risk pricing for these two instruments are related to both the uncertain revenue and the debt servicing.

The public sector, as the initiator of the PPP agreement, is considering valuation of real options as one of the possible strategies for the revenue risk mitigation. The private sector is interested in this financial instrument for the same reason, but also in other financial instruments like loans and bonds. The purpose of these instruments for the private sector is debt raising, which correlates it with financial institutions that are able to provide these forms of debt. In this setting, the proposed method is beneficial to all considered project participants.

As the final step, improvement actions in the network made by the public sector are introduced in the model. The sensitivity analysis of the identified financial instruments is performed for various marginal increments of network links and results are presented in following chapters. This step reveals the sensitivity of the revenue and financial instruments to the decisions made by the public sector regarding the increase of the capacity on links in the network. The analysis of results improves understanding of the effect that adding capacity to the network has on project's financial performance. It also provides an insight into the sensitivity of financial instruments providing the information which financial arrangements are more sensitive than others to the changes in the network structure.

SUMMARY

The research framework provided an insight into identification of key participants in toll roads agreements and their roles and interactions. Also, the value of the project is defined as the benchmark for the quantification of which network improvements will add the value to the toll road. Based on descriptions and explanations provided in the research framework, the evaluation method is developed as the decision-support tool. First step of the evaluation method, the network assignment process, is explained in detail in the next chapter.

CHAPTER IV

NETWORK ASSIGNMENT

Transportation users traverse the network at some costs. These costs can be expressed in multiple different ways. Typically, it is expressed as a travel time, but it also can be expressed as a cost of gas and toll, delay, or in any measure that the user evaluates when selecting their route (Meyer and Miller, 2001). Hence, the term generalized cost is often used. The total cost of the travel on some route between two points (origin and destination) is then equal to the sum of travel costs over used links on that route. Given a set of Origin-Destination (O-D) demand pairs, this concept is used to determine link flows through network assignment process, including the flow on a planned toll road link.

USER EQUILIBRIUM

Consider A as a set of all links in the network, U as a set of all O-D pairs and R as a set of all routes in the network. In general, cost function c_a of link $a \in A$ can be expressed as follows:

$$c_a = c_a^0 + c_a^t \quad (4-1)$$

where c_a^0 is travel cost on link a and c_a^t is equivalent cost value of the toll rate, if such exist on the link a .

The distribution of the traffic in a network is then determined according to Wardrop's first principle (Wardrop, 1952): *all used routes have equal travel time and all unused routes have the equal or a greater travel time*. In other words, users do not have incentives to change their route since they cannot find another route in the network with a lower travel time. The distribution of traffic in the network according to this principle is called User Equilibrium (UE).

Consider a link flow $V_a, a \in A$, demand between O-D pair $D_u, u \in U$ and set of routes between O-D pair \bar{R} . In mathematical terms, UE is a solution for the minimization of the objective function that satisfies the flow constraints. Objective function is a sum of the integrals of the link's cost functions, (Sheffi, 1984):

$$\min z(V_a) = \sum_{a \in A} \int_0^{V_a} c_a(\omega) d\omega \quad (4-2)$$

subject to:

$$\sum_{r \in \bar{R}} p_r = D_u \quad (4-2a)$$

$$\forall p_r \geq 0 \quad (4-2b)$$

$$V_a = A_{ar} * p_r \quad (4-2c)$$

where $p_r, r \in \bar{R}$ is a path between O-D pairs and A_{ar} is a link-path incidence matrix. Eq. 4-2a represents a constraint for preservation of all link flows, that is, the fact that all traffic demand needs to be distributed through the network. In addition, link flows cannot be negative (Eq. 4-2b) and they can be assigned only to routes which connect the observed O-D pair (Eq. 4-2c). In these terms, the link-path coincidence matrix A_{ar} has

elements of 0 and 1. If the observed link a is on the route p_r , then the element is 1 and 0 is otherwise.

Solving Eq. (4-2) with respect to the given constraints, one can determine traffic flows on all links in the network. However, the UE solution reveals traffic flow values for the given O-D demand which is here assumed to be deterministic. In other words, for the given O-D demand and known network topology, traffic flow on the toll link is calculated as deterministic. The next step is to assess and model the future link flow on the toll link.

FORECAST OF FUTURE TRAFFIC FLOW ON THE TOLL ROAD

The assumption made here is that the flow on the link of interest follows a stochastic process such as a geometric Brownian motion (GBM). This random process has been already used in the literature for this purpose (Brandao and Saraiva, 2008). For this research, properties of geometric Brownian motion and its application in this context are first assessed and evaluated.

Brownian motion is a stochastic process $\{X(t), t \geq 0\}$ with following properties (Karlin and Taylor, 1975):

- Every increment $X(t+s) - X(t)$ is normally distributed with mean μt and variance $\sigma^2 t$; μ, σ being fixed,

- For every pair of disjoint time intervals $[t_1, t_2], [t_3, t_4]$, where $t_1 < t_2 \leq t_3 < t_4$, the increments $X(t_4) - X(t_3)$ and $X(t_2) - X(t_1)$ are independent random variables with distribution defined previously, and correspondingly for n disjoint time intervals, where n is a positive integer.
- $X(0) = 0$ and $X(t)$ is continuous at $t = 0$.

However, since the every increment is normally distributed which allows that the underlying variable has negative values, it is not realistic to use this random process for the traffic flow modeling. The geometric Brownian motion (GBM) resolves this issue. GBM is a process defined as

$$S(t) = \exp[X(t)], t \geq 0 \quad (4-3)$$

First two moments of this process are

$$E[S(t) | S(0) = s] = s \exp\left[t\left(\mu + \frac{\sigma^2}{2}\right)\right] \quad (4-4)$$

$$Var[S(t) | S(0) = s] = s^2 \exp\left[2t\left(\mu + \frac{1}{2}\sigma^2\right)\right] [\exp(t\sigma^2) - 1] \quad (4-5)$$

Defining the traffic flow as geometric Brownian motion allows the modeling of traffic as an uncertain process which evolves over time stochastically. From Eq. (4-4) and Eq. (4-5), at each time segment t , traffic flow can be defined with lognormal distribution with the known expected value and variance. Thus, uncertainty in the future traffic flow on the toll road can be modeled with the lognormal distribution with known parameters.

The GBM has two properties that were found very useful in this research:

- Expected traffic increases at some constant rate,
- Traffic flow at the time $t + s$ depends only on the traffic flow at time t regardless of previous states.

The first property states that the expected traffic flow on the observed link increases at some constant rate. From the literature review, it has been noted that the traffic forecast are not accurate and that the prediction of the single rate at which the traffic will increase, which is used in practice, is not the best approach. The second property relaxes this assumption.

The second property is known as the memoryless property of a stochastic process. The underlying variable does not have any ‘memory’ about previous states. In the context of traffic flows, as it was used in this thesis, this can be explained as follows: network users have a choice in their trip origination to decide between available means of transportation or available routes. This choice will depend on the information that users have obtain in the previous time state. Indeed, information about traffic conditions is almost instantly available to users, especially with the development of new technologies. Thus, users choice of routes or means of transportation is time adjustable as the new information becomes available.

Also, following the Yescombe’s discussion (2002), if the traffic grows above expected projection, this is probably not because of the project, but more the result of the overall growth in the economy. Similarly, if the traffic is below projection, it can be

assumed that this might occur due the economy downturn. Hence, the realization of the traffic on the toll road might vary over time due to eternal factors which are out of project participants control. Thus, it is realistic to assume that the realization of the traffic flow in the next time frame will depend only on the current realization. After this analysis of GMB and its application in the context of this research, it is adopted to model the traffic flow as the stochastic process.

Thus, the link flow is defined as follows:

$$dV_a = \mu V_a dt + \sigma V_a dW_t \quad (4-6)$$

where V_a is the starting value of traffic flow on the toll link obtained from traffic assignment, μ is a drift rate, σ^2 is a variance, $dW_t = \sqrt{dt}\varepsilon_t$ is a Weiner process where dt is a time increment and $\varepsilon_t \sim N(0,1)$. The future traffic flow can be fully defined by knowing its starting value V_a , the expected growth rate μ and the volatility σ . Irwin (2003) suggests that the values for the growth rate and the volatility of the revenue process can be extracted from the past data, if such is available, or from similar projects. Here, it is assumed that these two values for the traffic flow can be derived from similar projects and that they are constant over the concession period.

STOCHASTIC REVENUE PROCESS

The traffic flow on the link of interest is modeled as the geometric Brownian motion. Knowing that the revenue depends on the traffic flow and the toll rate, it can be

defined that the yearly revenue is equal to the yearly traffic on the toll link and the toll rate. Then, the revenue function R can be defined as follows (Brandao and Saravia, 2008):

$$R = V_a * T_r \quad (4-7)$$

assuming that V_a is expressed as the annual traffic. From Eq. (4-6) and Eq. (4-7), since the revenue function is expressed as a function of the link flow $f(V_a)$, and based on Ito's lemma which is defined as (Trigeorgis, 1998):

$$dR = \frac{\partial R}{\partial t} dt + \frac{\partial R}{\partial V_a} dV_a + \frac{1}{2} \frac{\partial^2 R}{\partial V_a^2} (\sigma^2 V_a^2 dt) \quad (4-8)$$

then the revenue can be defined as the process which evolves over time stochastically:

$$dR = \mu V_a T_r dt + \sigma V_a T_r dW_t \quad (4-9)$$

The parameters which determine the behavior of the revenue over time are: starting annual traffic flow on the toll link V_a , expected traffic growth rate μ , volatility σ and the toll rate T_r .

NETWORK EXAMPLE

To illustrate the developed model and the first step, the network assignment, consider a simple unique path network example illustrated in Fig. 4-1 (Damnjanovic *et al.*, 2008). The network is defined with only one O-D pair and four links with three unique paths. Link 2 is a toll road link; link 1 is clearly feeder link, while links 3 and 4

are competing. D_{A-B} represents traffic demand between node A and node B. The first path p_1 is consistent of two links: link 1 and link 2; the second path p_2 has two links also: link 1 and link 3, while third path p_3 has only one link, link 4. All links are labeled with the link cost function c_i and the link flow $V_i, i = 1, 2, 3, 4$. This network, with given one O-D pair, allows easy following of the impact that change in the network structure (i.e. link's capacity) will have on the distribution of traffic flows under deterministic O-D demand. Also, this network allows easy and straightforward categorization of feeder and competing links with the respect to the toll link since the methodology for identification of these links is not explained in this thesis, that is, it is not part of this thesis.

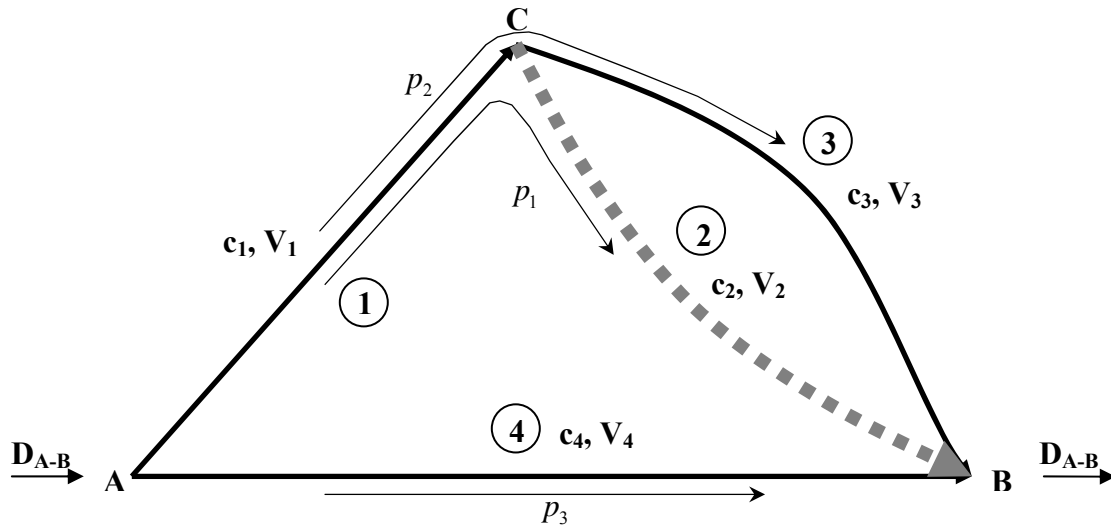


Fig. 4-1. Network example

As mentioned earlier, distribution of O-D demand in the network is determined based on Wadrop's first principle, User Equilibrium. In order to derive the solution for the objective function (Eq. 4-2), cost function of link's performance first needs to be specified. Generalized Bureau of Public Roads (BPR) link performance function is used:

$$t_a = t_a^0 \left[1 + \alpha \left(\frac{V_a}{C_a} \right)^\beta \right] + c_t, a \in A \quad (4-10)$$

where t_a^0 is the free flow time on link $a \in A$, α and β are model parameters and C_a is the link capacity. Values for parameters $\alpha = 0.15$ and $\beta = 4$ are usually used for the calculation process (Sheffi, 1984). However, these values indicate that the capacity of the link is equal to the traffic flow at travel time which is higher than the free flow travel time by 15 percent. Assuming that the capacity of the road is higher, parameter α is set to 0.5 implying that the link's capacity is equal to the traffic flow at the travel time which is higher than the free flow travel time by 50 percent. Also, parameter β is set to 1 in order to obtain the linear cost function, so the closed form solution for link flows can be obtained. Thus, the following two parameters can be defined: $a_a = t_a^0$ and $b_a = (t_a^0 * \alpha) / C_a$. Note that the parameter b_a is a function of a link's capacity C_a . Assuming that the BPR link performance function can be expressed with the equivalent cost of time, Eq. 4-10 becomes:

$$c_a = a_a + b_a * V_a + c_t, \quad a = 1, 2, 3, 4 \quad (4-11)$$

The parameters of the link performance function are summarized in Table 4-1. Demand between A and B is $D_{A-B} = 50,000$. Units are excluded for the purpose of

generalizing results, but this can be easily relaxed to incorporate different conditions (capacity of the link, time equivalent of the toll rate and the link's free flow time). Link 1 is considered a feeder link or the link that has a positive correlation with the toll link. Any increase in the capacity on the feeder link will positively affect the flow on the toll link, and vice versa. Alternatively, the impact of the competing link is negative; increase in the capacity on the competing link decreases the flow on the toll road, and vice versa. From Fig. 4-1, link 4 can be clearly distinguished as the competing link since it is on the path that competes with the toll link 2. Also, link 3, which belongs to the path 2, is declared as the competing link.

Table 4-1. Summary of links parameters

Link No.	a_a	C_a	c_t
1	10	2500	
2	6	2000	0.15
3	6	2500	
4	8	3000	

The solution that satisfies the UE condition is a solution to the problem defined in Eq. (4-2). If the objective function is expressed in terms of paths flows, then the minimization problem becomes:

$$\begin{aligned} \min z(p_j) = & \int_0^{p_1+p_2} (a_1 + b_1 * \omega) d\omega + \int_0^{p_1} (a_2 + b_2 * \omega + c_t) d\omega + \\ & \int_0^{p_2} (a_3 + b_3 * \omega) d\omega + \int_0^{p_3} (a_4 + b_4 * \omega) d\omega \end{aligned} \quad (4-12)$$

subject to:

$$\sum_{j=1}^3 p_j = D_{A-B} \quad (4-12a)$$

$$\forall p_j \geq 0 \quad (4-12b)$$

where $j = 1, 2, 3$ is the number of paths for a given O-D pair D_{A-B} .

The flow on the toll road V_2 (path p_1) can be found by solving the Lagrangian (Sheffi, 1984):

$$L(p_j, \lambda) = z(p_j) + \lambda^* (D_{A-B} - \sum_j p_j) \quad (4-13)$$

subject to:

$$p_j \geq 0, j = 1, 2, 3 \quad (4-13a)$$

Solving Eq. (4-13), closed form solution for all path flows is derived as follows:

$$p_1 = \frac{\lambda - a_1 - a_3}{b_1} - \frac{b_1 + b_3}{b_1} \frac{\lambda b_1 - b_1(a_1 + a_2 + c_t) - (b_1 + b_2)(\lambda - a_1 - a_3)}{b_1^2 - (b_1 + b_3)(b_1 + b_2)} \quad (4-14)$$

$$p_2 = \frac{\lambda b_1 - b_1(a_1 + a_2 + c_t) - (b_1 + b_2)(\lambda - a_1 - a_3)}{b_1^2 - (b_1 + b_3)(b_1 + b_2)} \quad (4-15)$$

$$p_3 = \frac{\lambda - a_4}{b_4} \quad (4-16)$$

where

$$\lambda = \frac{[D_{A-B} b_1 b_4 + b_4(a_1 + a_3) + b_1 a_4] [b_1^2 - (b_1 + b_3)(b_1 + b_2)]}{(b_1 + b_4) [b_1^2 - (b_1 + b_3)(b_1 + b_2)] + b_2 b_3 b_4} + \frac{b_3 b_4 [(a_1 + a_3)(b_1 + b_2) - b_1(a_1 + a_2 + c_t)]}{(b_1 + b_4) [b_1^2 - (b_1 + b_3)(b_1 + b_2)] + b_2 b_3 b_4} \quad (4-17)$$

and link flows are $V_1 = p_1 + p_2$, $V_2 = p_1$, $V_3 = p_2$ and $V_4 = p_3$.

Having the link flow on the toll link determined, next step is to forecast the future traffic in order to model the uncertainty in the future flow outcome. Since it is assumed that the traffic flow follows over time the stochastic process, the link flow is defined as:

$$dV_2 = \mu V_2 dt + \sigma V_2 dW_t \quad (4-18)$$

Values for growth rate and volatility of the traffic flow are assumed to be constant over time. Then, the process defined in Eq. (4-18) is determined for the given time of the concession period. Further, the revenue is equal to $R = V_2 * T_r$ and thus, is also a stochastic process:

$$dR = \alpha V_2 T_r dt + \sigma V_2 T_r dW_t \quad (4-19)$$

Monte Carlo method is used for a simulation of revenue values for the project's operational life.

Standard Error

If θ represents a true value of the estimate, a random variable Θ with the distribution $(\mu_\theta, \sigma_\theta^2)$ and $\theta = E[\Theta]$. If the number of simulations is N and $\Theta_n, n \in N$, then the mean value $\hat{\Theta}$ is the arithmetic average:

$$\hat{\Theta} = \frac{1}{N} \sum_{n \in N} \Theta_n \quad (4-20)$$

According to the central limit theorem, sample distribution of the sample mean $\hat{\Theta}$ is approximately normal with a mean μ_θ and variance σ_θ^2/N .

Value $\hat{\Theta}$ is the point estimator for the value θ and it is unbiased if the $E[\hat{\Theta}] = \theta$, and biased if the $E[\hat{\Theta}] = \theta + error$. Standard error of the estimator $\hat{\Theta}$ is its standard deviation:

$$SE(\hat{\Theta}) = \sqrt{V(\hat{\Theta})} = \frac{\sigma_{\theta}}{\sqrt{N}} \quad (4-21)$$

From the last equation, if the number of simulation is increasing, then the standard error of the estimator $\hat{\Theta}$ decreases.

The standard error for the revenue value depends on the number of simulations and the variance. Since the variance of the revenue modeled as the geometric Brownian motion (Eq. 4-19) depends, among other parameters, on the time over which this stochastic process is simulated. Thus, the standard error for the revenue is reported in the following chapters in sections that are presenting results of sensitivity analysis.

SUMMARY

Network assignment process provided the solution for the traffic flow on the toll link for the given network and the given O-D demand. Geometric Brownian motion is introduced as the process for the modeling of future traffic flows on the toll link and associated revenues. Analysis of financial instruments is introduced in following chapters starting with the bank loans.

CHAPTER V

BANK LOANS

Bank loans are widely used in PPP agreements as the financial instrument for raising the debt. These loans are suitable for the negotiation phase when the project's revenues can only be estimated and uncertain because, in the case if the project has difficulties in early stages of operation phase, banks are willing to renegotiate terms of the loan. The quality and the strength of the project to service the debt is estimated from different parameters, including the main risk in PPP agreements – the revenue risk. The approach and the methodology that banks use for the risk assessment in project finance agreements are addressed in this chapter. In addition, the identified key risk parameters are correlated with the uncertain revenue model used in this thesis as the part of the developed method for financial analysis of toll roads.

KEY RISK PARAMETERS

As previously mentioned, the new Basel Accord (Basel Committee on Banking Supervision, 2005) recognizes probability of default (PD), exposure at default (EAD) and losses given default (LGD) as key risk parameters in credit risk analysis. In addition to these three risk parameters, for the calculation of the loan's risk weight, banks need to forecast the level of potential credit losses. For this purpose, the new Basel Accord defines the distribution of real losses as the necessary input for the assessment of

expected losses (EL) and unexpected losses (UL). EL represents the mean of the assessed distribution of losses and different bank's provisions cover them. However, banks also need to have covered the certain percentile of the assessed real losses (99.9th percentile) and the difference between this threshold value and EL is considered to be UL. It is assumed that, in times when the economy has a downturn, banks will be covered with the 99.9% of all potential losses. In other words, banks will have the probability of 0.1% to become insolvent.

Probability of Default

As has been noted, it is assumed that the default occurs when the borrower defaults on his debt obligations and stays below debt level until the end of the loan life. In other words, when the revenue generated from the project drops below the debt repayment level and its projected path remains under the debt obligations level, the project is considered to be in default. In the developed modeling framework, the probability of default is defined based on the event of toll road revenue process that follows GBM hitting a barrier D_l , where barrier is below starting revenue $D_l < R(0) = r$, during loan life T . The probability that the revenue process will hit the barrier D_l at time interval $(0, T)$ and stay below D_l until the end of concession life T (e.g. project defaults) can be defined as:

$$P\{R(t) \leq D_l \cap R(T) \leq D_l\}, 0 \leq t \leq T \quad (5-1)$$

Real Losses

However, a default event does not imply complete losses. To determine the extent of the losses, exposure at default (EAD) and real losses need to be assessed. Exposure at default (EAD) represents the amount of loan at risk if the default occurs. Again, this does not necessary imply that the whole amount will be lost as there is significant “residual value” of the project. This is illustrated in Fig. 5-1. The project’s revenue has been dropped below debt service D_l in year t_l . The bank has invested some amount that is scheduled for repayment at some constant rate D_l . Until the default point, the lender (bank) has received t_l repayments totaling $D_l t_l$, EAD is then:

$$EAD = D_l (T_l - t_l) \quad (5-2)$$

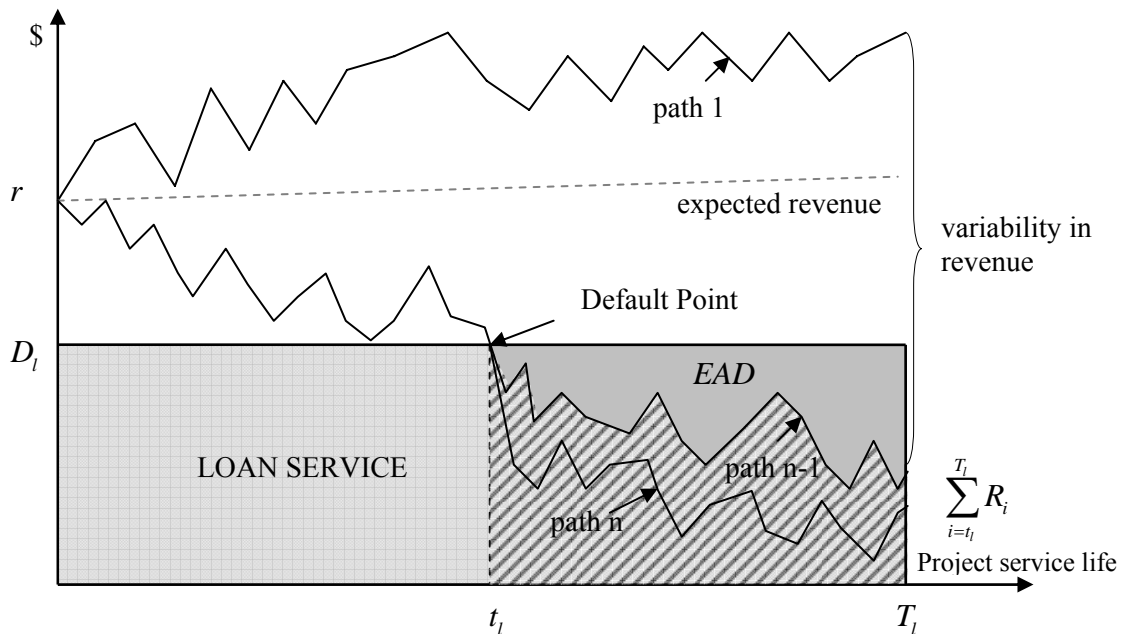


Fig. 5-1. Default Point for loans

The loss given default (LGD) is a ratio between the real losses and the EAD. When the default occurs the bank is exposed at the level of EAD. As the project continues to generate revenue over the remaining time horizon $(T_l - t_l)$, the bank recovers some of the losses. The amount that the bank can recover from continuation of the project's operation is a sum of the remaining revenue. The real losses (RL) are then the difference between risk (EAD) and the amount that the bank can recover:

$$RL = EAD - \sum_{i=t_l}^{T_l} R_i \quad (5-3)$$

From this equation, LGD can be expressed as:

$$LGD = \frac{RL}{EAD} = 1 - \frac{\sum_{i=t_l}^{T_l} R_i}{EAD} \quad (5-4)$$

Nevertheless, remaining revenues are uncertain since the revenue behaves stochastically over time. If the project defaults, based on the previous definition of default, it is uncertain how much the bank can recover from continuation of the project's operation. For example, using the Monte Carlo simulation, if the number of simulated paths for yearly revenues over project's service life is n and number of simulated paths below debt service is m , then the distribution of real losses can be determined for the sample size of m paths as the difference between EAD and the sum of revenues for those paths (Eq. (5-3)). The mean of that distribution is EL and 99.9th percentile is the threshold level or Value at Risk (VaR).

Risk Weighted Assets

New accord, Basel II, distinguishes two parts of potential losses given default: Expected Losses (EL) and Unexpected Losses (UL) (Basel Committee on Banking Supervision, 2005). EL are losses which banks expect to occur and thus are covered with different provisions. EL represent the average level of potential losses. In order to derive a corresponding asset weight, Basel II recommends an upper level 99.9% confidence interval for losses to be covered by capital reserves. The value of loss expected to be exceeded with some probability (i.e. 0.1% for this case) is known as the Value-at-Risk (VaR) (Linsmeier and Pearson, 2000). The difference between VaR and EL represents UL.

Basel II requires that banks provide capital requirements that will buffer UL at the VaR level. Capital requirements are set to be minimum 8% of UL (Basel Committee on Banking Supervision, 2004).

$$CAR = \frac{\text{Regulatory Capital (Tier I)}}{\text{RWA}} \quad (5-5)$$

where CAR is a Capital Adequacy Ratio (equal to 8%) and RWA is Risk Weighted Assets (Basel Committee on Banking Supervision, 2005).

Without loss of model generality, the following assumptions are used to calculate RWA: the loan is not correlated to any systematic risk factor and maturity adjustments are not applicable. Basel II takes into account asset correlation to the systematic risk factor for the capital requirement calculation (Basel Committee on Banking Supervision, 2005). However, the underlying variable for the credit risk assessment of the project finance loans is the revenue. Revenues from toll roads can not be considered as assets

nor are they traded on the market (Irwin, 2003). Thus, the correlation of the loan, which depends on the estimated project's cash flows (revenues from toll road) with systematic risk factor, is not taken into account.

Similarly, the maturity adjustment in the calculation of the capital requirement addresses the issue of potential downgrades over long-term credits. These downgrades represent the probability that borrowers will move from one rating grade to another, or, in other words, that the probability of default will change over given time horizon. However, as mentioned above, the assessment of credit risk for the revenue-generated projects is related only to the revenues. Uncertainty in these cash flows is already taken into account since they are modeled as stochastic process. Probability of default is increasing over time as the uncertainty in revenues is increasing over time. Changes in probability of default are already captured, thus, the maturity adjustment is also not taken into account.

Credit worthiness is related only to the forecasted cash flows hence financial viability is fully captured by traffic and revenue models. EL is derived as the expected value of real losses distribution. Similarly, VaR is determined at the 99.9% confidence level. UL are the difference between EL and VaR (Basel Committee on Banking Supervision, 2005):

$$UL = VaR - EL \quad (5-6)$$

where EL is a mean of a distribution of real losses given default and VaR is 99.9th percentile of the same distribution. The capital of the bank is set to cover the gap between EL and VaR (Basel Committee on Banking Supervision, 2005). In order to

calculate RWA, the gap needs to be multiplied by a factor 12.5 (reciprocal of 8% minimum capital requirements ratio):

$$RWA = 12.5 * PD * UL \quad (5-7)$$

where PD is a probability that the project will default in given year and UL are unexpected losses derived from the distribution of real losses for default in given year. Here, it should be noted that UL are derived from the distribution of real losses given that the project has default in given year. Hence, the distribution of real losses is conditional on default.

Bank's risk parameters are defined based on uncertain future cash flows solely since this property is specific for project finance agreements. The sensitivity of parameters to changes in the network structure is presented in the following section on the simple network example.

SENSITIVITY ANALYSIS

The set of input values for Monte Carlo simulation is presented in the Table 5-1. Monte Carlo simulation is used for the simulation of paths for revenue given its initial value $R(0)$. This value is determined from the UE solution for the given network. Values for the revenue drift rate μ and the volatility σ are assumed and these values are adopted from the literature. Drift rate of 5 percent represents the expected traffic growth rate and the volatility of 20 percent represents the standard deviation of the growth rate. Sorge and Gadanecz (2004) report the average maturity for project finance

loans of 12 years for sample of loans with public sector guarantees and 7.5 years for sample of loans with and without guarantees. Thus, in this research, the assumed length for debt service T is chosen to be 10. Since the toll rate is usually set per mile and, in this network example, the length of all links is not discussed, the average toll rate T_r is set to 1 for the simplicity of calculation. The ratio between the starting revenue $R(0)$ and the debt level D_l is set to be 1.4 as the level of annual debt service cover ratio for standard toll road projects (Yescombe, 2002). The number of simulated paths for the revenue process is $n=100,000$ and standard error for the revenue simulation is $SE=6.2*10^3$. The total value of debt scheduled for the repayment over time T is $16.8*10^6$.

Table 5-1. Summary of input parameters for Monte Carlo simulation for loans

T	$R(0)$	μ	σ	T_r	$\frac{R(0)}{D_l}$	n
10	$2.35*10^6$	0.05	0.2	1	1.4	100,000

From the Monte Carlo simulation, the distribution of real losses is assessed given that the project defaulted, based on the definition of default used in this research. The distribution of real losses is determined for the given network structure parameters (Table 3-1) and simulation input parameters (Table 5-1) and these sets are considered as the base case. This base case represents results of the proposed methodology for the assessment of bank's risk parameters under uncertain traffic demand. Further, changes in

the network structure are introduced as changes in links capacities, thus changes in the parameter b_i (Eq. (3-8)). The sensitivity analysis of bank parameters provides an insight in the phenomena of the impact that different public sector decisions regarding the network improvements have on bank parameters and bank's capital structure.

Distribution of Real Losses Given Default

As previously mentioned, real losses are determined for the paths that have crossed the lower barrier (e.g. below debt repayment level in year 1 and remained below this level during the loan life). Based on the definition of default in this paper, this case represents the default of the project in 1st year of debt service. In other words, given that the project has started the debt service with the debt cover ratio of 1.4, there is a probability of default in the 1st year of service as the consequence of the uncertain traffic demand. It is assumed that the lender cannot fully recover the outstanding debt, and the loan is considered to be in loss. Fig. 5-2 illustrated distribution of real losses. The horizontal axis indicates the total losses, while the vertical axis represents the number of simulated paths below the debt service.

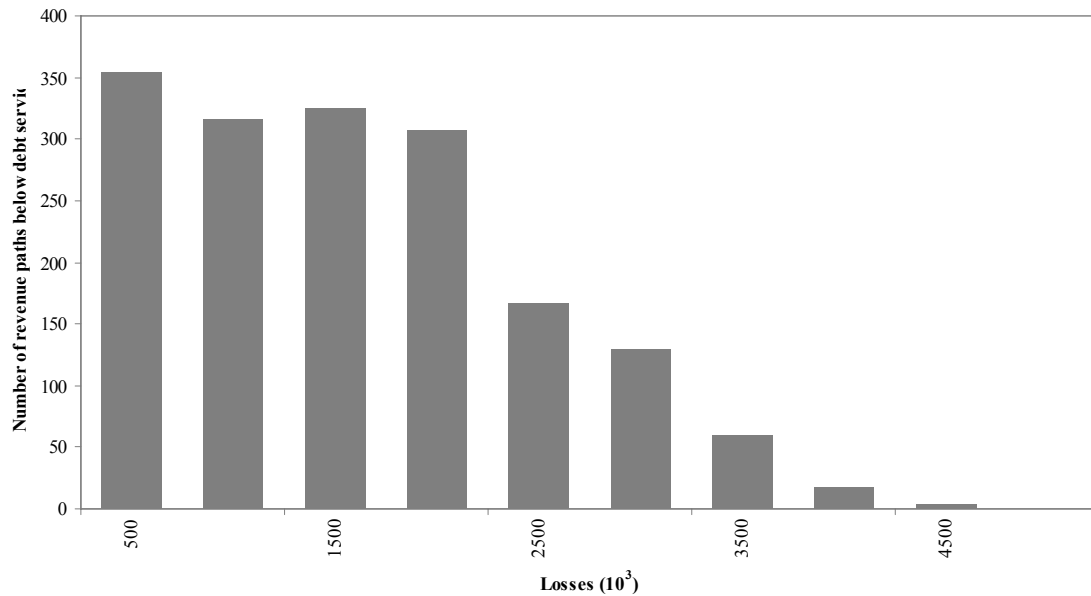


Fig. 5-2. Distribution of real losses given default in 1st year of debt service

For this baseline scenario, the probability that the project will default in 1st year is estimated to be 3.3%. The expected losses EL and Value-at-Risk at 99.9 percent level are estimated to be $1,340 \cdot 10^3$ and $4,290 \cdot 10^3$, respectively. The unexpected losses UL are $2,950 \cdot 10^3$ and associated risk weight of an asset RWA is $1,220 \cdot 10^3$. These values are calculated parameters for bank's risk assessment based on the proposed method and given network structure under uncertain traffic flow. In the following section, changes of the links capacity are introduced as the measure that can affect financial variables and financial performance of a project in general.

Sensitivity of Financial Parameters

The stress analysis is conducted to examine the effect of changes in the network structure on financial variables. The stress analysis considers marginal changes in link's capacity at predefined time period. As previously mentioned, in the network structure illustrated in Fig. 4-1, link 1 is a feeder link, while link 3 and link 4 are competing links. The capacity values are varied from $C_{\min}=1,500$ veh/time unit to $C_{\max}=3,500$ veh/time unit with the marginal increment of $\Delta C=50$ veh/time unit on links 1 and 3. These two links are chosen in order to show effects of changes on the feeder versus the competing link respectively.

It is expected that the increase in the capacity of the feeder link will positively affect risk parameters, thus add to the value of the project. Probability of default and risk weight of the asset should decrease as the travel cost on the feeder link decreases, thus attracting more traffic to it. As the number of trips on feeder link increases, it will also increase the traffic on the toll link. Thus, the higher the number of trips on the toll link, the lower the probability it will default on its debt service. In contrast, the increase in the capacity on the competing link will detract from the value of the project.

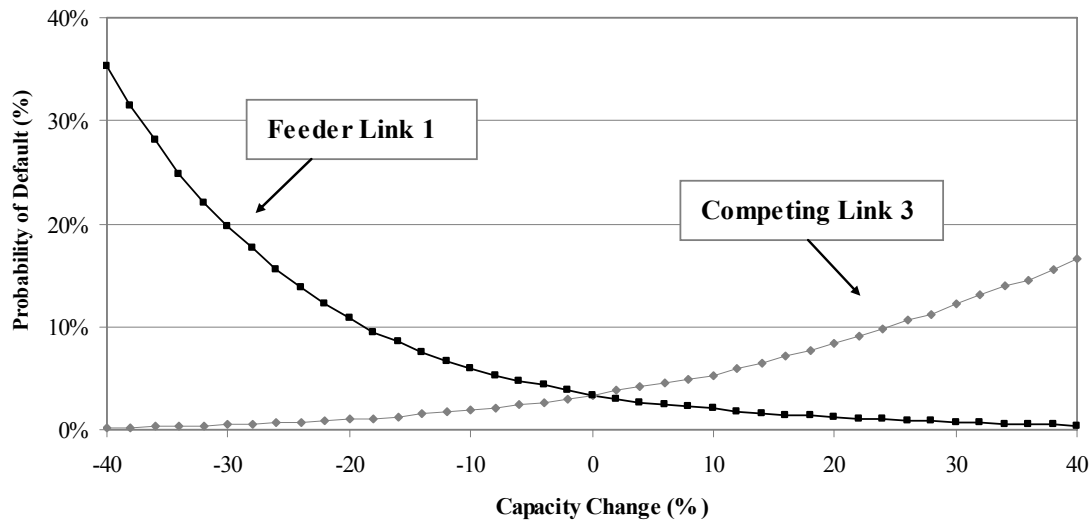


Fig. 5-3. Changes in a probability of default for a 1st year of debt service

Fig. 5-3 illustrates the dependency between the probability of default and the change in capacity on feeder link 1 and competing link 3. The horizontal axis represents the percent change of the link's capacity and the vertical axis represents the probability that the project will default within 1st year of debt service. As the capacity of feeder link increases, the probability that the project will default on its debt obligation decreases, as it was expected. For competing link 3, the result is reverse. As the capacity of the competing link increases, the probability that the project will default increases as well. Larger the capacity of the competing link, the more traffic will be attracted to that link instead to the toll link.

As it can be observed from Fig. 5-3, this relation between probability of default and capacity change is nonlinear. This means that the probability of default will depend on the size of the capacity change and the starting value, (e.g. starting capacity of the

link). This is an important implication as it indicates that an initial network structure should be measured as strategic consideration when developing projects. Further, the change in the probability of default for the competing link and the feeder link is not symmetrical. Observing the results from Fig. 5-3, which are related to the increase in the probability of default, it can be concluded that decrease in the capacity on the feeder link has the higher impact than the increase in the capacity on the competing link. The vice versa is not that obvious from results implying that the same decrease in the probability of default can be achieved either by decreasing the capacity on the competing link or increasing the capacity on the feeder link. The impact these changes might have on the probability of default, for this case study, appears to be almost the same. In other words, given incremental increase in the capacity on the feeder link has the same impact as the same incremental decrease in the capacity on the competing link.

Fig. 5-4 illustrates the effect of change in network structure on risk weights. Similarly with previous figure, horizontal axis is the change of the link's capacity expressed as the percentage and vertical axis is the percent change of the risk weight of the asset. The results are intuitively similar to the one obtained for the probability of default. For example, an increase in the capacity on the feeder link will lead to the decrease in loan's risk weight. On the other hand, an increase in the capacity on the competing link will result in an increase in risk weight. This presentation of network to loan risk weight is more direct and understandable to lenders.

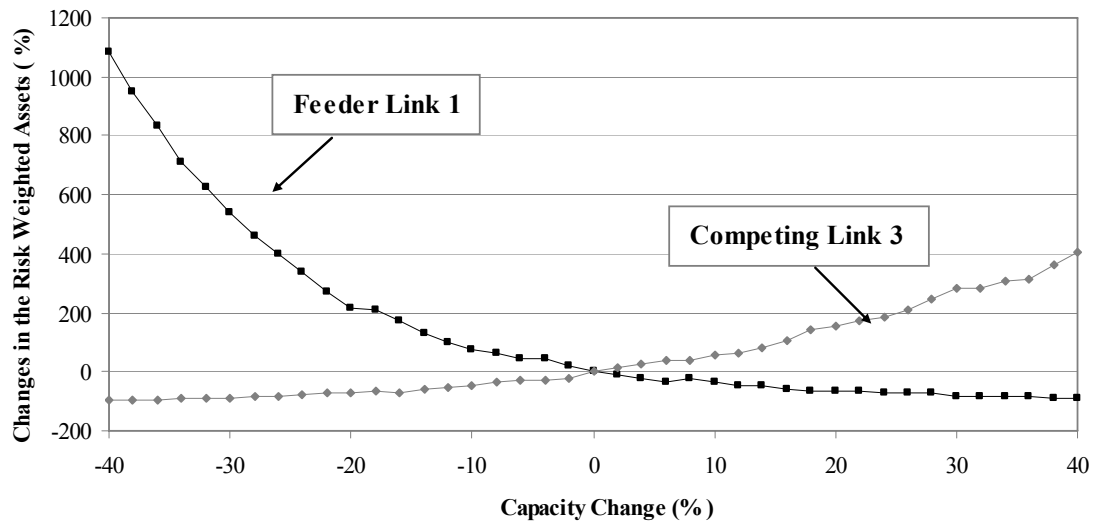


Fig. 5-4. Changes in a risk weight for a 1st year of debt service

Similarly with the interpretation of results for probability of default, the incremental change in the risk weight of the asset is nonlinear and asymmetric. Observing the increase in the RWA, the marginal decrease on the feeder link has higher impact than the marginal increase in the capacity on the competing link. The opposite changes on the competing and the feeder link appears to have the same impact on the decrease of RWA. In order to gain insight into the magnitude of these effects, results of the sensitivity analysis are presented in Table 5-2.

Table 5-2. Summary of results for bank loans sensitivity analysis

Capacity Change		Δ Probability of Default (1 st year)	Δ Risk Weighted Assets
Feeder Link $\Delta C1$	+10%	-40%	-45%
	-10%	+77%	+97%
	+20%	-66%	-60%
	-20%	+227%	+241%
Competing Link $\Delta C3$	+10%	+58%	+54%
	-10%	-43%	-45%
	+20%	+143%	+139%
	-20%	-71%	-71%

Decrease in the PD occurs when the capacity on the feeder link is increased or if the capacity on the competing link is decreased. For example, if the capacity on link 1 is increased by 10%, the PD will decrease by 40%. The opposite marginal change on link 3 will decrease the PD by 43%. If the magnitude of these changes rises to $\pm 20\%$ than, for the change on link 1, the PD will decrease by 66%, and, for the change on link 3, the PD will decrease by 71%. Similar results can be obtained for the changes in the RWA. Although there are differences in the magnitude of the change both for the PD and RWA, results indicate that these values are close to each other.

In contrast, the increase in the PD appears when the capacity increases on the competing link or decreases on the feeder link. Increase in the capacity of the competing link 3 by 10% will increase the PD by 58%. If the capacity on the feeder link 1 is decreased by 10%, it will increase the PD by 77%. Similarly, if these marginal changes on capacities rise to 20%, the impact on the PD significantly alters. The increase in the

PD for the change on the link 3 is 143% while for the change on the link 1 is 227%. The impact is thus significantly different for network improvements actions on the feeder link 1 and the competing link 3.

SUMMARY

In this chapter, key risk parameters, which occur in banks regulation, are analyzed in the context of toll roads. The revenue risk is used as the primary risk and the risk parameters are defined based on the properties of the GBM. Monte Carlo simulation is used for the assessment of these bank's parameters and for the sensitivity analysis. Results are obtained for the simple network example in which the feeder and the competing link can be intuitively identified. The results have shown that the decrease in the capacity on the feeder link has greater impact on risk parameters than the increase in the capacity on the competing link. In the following chapter, the identification of key risk parameters for the next financial instrument, bonds, is presented. Financial risk for bonds is also correlated with the revenue risk and the sensitivity analysis is conducted.

CHAPTER VI

BONDS

As discussed earlier, there are several approaches to model and price the credit risk. The common underlying goal in all these models is to forecast the probability of default based on some assumptions. For example, in the credit migration approach, the probability that the borrower will move from one credit grade to another including default is assessed from the value of the borrower's portfolio (Crouhy et. al., 2000). The initial step is to assign the rating grade to the borrower and then to estimate probability of moving from one grade to another. This initial rating grade can be accepted based on the rating of rating agencies like Moody's, Standard & Poor and Fitch.

The actuarial approach is based on the modeling of default as the Poisson process. There are no assumptions on why the borrower defaulted and the capital of the firm is not related to the default risk. The input in this model is a mean of Poisson distribution, or expected number of defaults within given time horizon. Next model, CreditPortfolioView, is based on the estimation of macro-variables which are specific for each country. The probability of default is conditional on these macro-variables and it is modeled as a logit function.

Structural approach is based on the asset value or the market value of the firm. It is assumed that the value follows the geometric Brownian motion and it is equal to the sum of future cash flows (Giesecke, 2004). Probability of default occurs when the value drops below the debt level. Here, two situations should be distinguished. First, when the

value drops below the debt level and remains below for the debt service life, it is considered that the obligator has defaulted. This approach to the definition of default is called classical approach. First-passage approach defines the default when the value drops below the debt level for the first time. It does not observe the remaining debt service and the behavior of the value over remaining time.

Each of these four models was analyzed in order to examine its application for the pricing of credit risk for toll roads. Some limitations of these models were observed for the intended application. First, the toll road is a stand alone facility which value is derived from its forecasted revenues. Modeling revenue as geometric Brownian motion captures the uncertainty in the future outcome and, thus, the probability of default can be assessed for the whole concession life. Thus, the approach of assessing the probability that the value of the toll road, that is, the sum of the forecasted cash flows, will move from one credit grade to another is not applicable. Also, modeling the probability of default as Poisson process does not apply for this model. As mentioned, the GBM captures uncertainty thus allowing the variation in the traffic demand and revenue over time.

However, macro-variables like unemployment do affect traffic demand, and these factors are already included in the traffic studies which are prepared before the concession agreement is offered. Based on different factors like unemployment or land development, expected traffic growth rate will be assessed. For these reasons, structural approach is adopted as the model for the assessment of the credit risk for toll roads. The

key risk parameters of this model and its application within this research are discussed in the following section.

KEY RISK PARAMETERS

Main principles in the structural approach for the credit risk assessment date back to Merton (1974). First, the value of the firm is a stochastic variable over time and it is the main risk driver of the credit risk. Also, an asset can be traded instantaneously and the debt liability is a claim on the asset. Main risk parameter is the probability of default or the probability that the obligor will default to repay the debt. Also, in order to price the credit risk, the price of the risky debt is compared with the price of the risk free debt. The difference between rates of return on these two debts, risky and risk free, is a credit spread.

Application of this approach to the pricing of the revenue risk in toll roads required some assumptions. First assumption is that the revenue, modeled as the stochastic process and the source for the debt repayment, is the claim for the debt liability. This means that, in the case of default, the bond investor will receive remaining revenue collected from the continuation of the road service. Second, the default occurs when the revenue process drops below the debt level and remains below for the debt service life (classical approach). This definition of default is the same as in the analysis of bank loans (Chapter V). Since the probability of default was already analyzed in the

Chapter V, following section explains in detail the model for the risk pricing, that is, the model for the assessment of the credit spread.

Credit Spread

Investors in a bond market will observe the credit spread of an offered bond as parameter for their return on investment and associated risk. Investor makes a decision based on the risk and adequate compensation which integrated in a bond price about purchasing the bond or not. Yield is the actual return on the bond that investor will earn. For example, if the bond is selling for a price P with a face value K and a coupon payment C and has a maturity T , then from equation:

$$P = \sum_{t=1}^T \frac{C}{(1+r)^t} + \frac{K}{(1+r)^T} \quad (6-1)$$

r is the rate of a return for the investor, or the yield.

Credit spread is a difference between the yield of a risky bond and the yield of a risk free bond. In theory, it can be assumed that there exist a risk free bond and usually, practitioners use the yield of a government or a treasury bond with similar maturity for the calculation process. Risky bond can be defined as the bond that has some probability of default. Same as the definition above, the probability of default is the probability that the bond issuer will fail on the repayment of promised coupon rate or principle at the maturity. Price of the bond is determined in such a way that includes this risk. In other words, price of the risky bond will be lower than the risk free bond because of the reduction caused by the price of the associated risk. The yield of the risky bond is higher

than the yield of the risk free bond. The explanation of the pricing method for the revenue-generated projects is presented in Fig. 6-1.

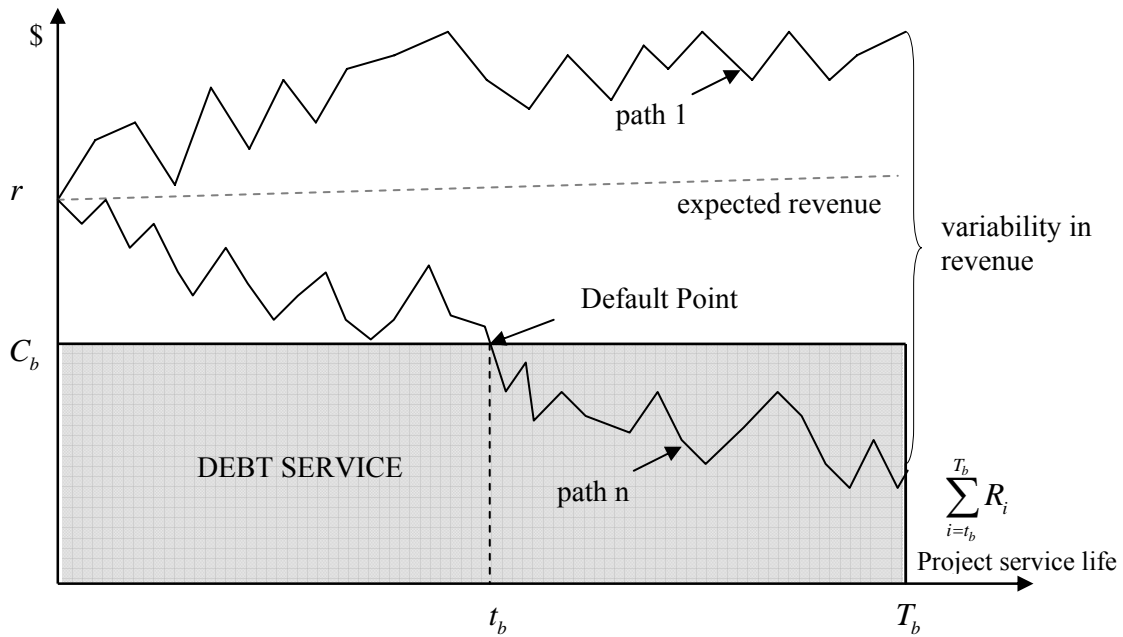


Fig. 6-1. Default point for bonds

Total amount of debt that is planned to be raised is defined as D_b . Further, it is assume that the loan is planned to be repaid through equally yearly repayments C_b and the debt life is T_b . If the revenue R is higher than the C_b for the whole debt service T_b , than the bondholder receives all promised payments. If R drops below C_b during the debt service at some time $t_b \in [0, T_b]$, bondholder will receive remaining revenue and it will experience a lost of:

$$C_b(T_b - t_b) - \sum_{i=t_b}^{T_b} R_i \quad (6-2)$$

This position of the bondholder is the same as he had sold the American put option with the strike price $C_b(T_b - t_b)$ and maturity T_b . American option is a type of the option that can be exercised at any time of option's life. This is equivalent to the definition of the default as it can occur at any time of the debt service. Further, put option is a type of the option that is used for the risk hedging when the price of the asset is expected to decrease. The seller of the put option accepts the obligation to buy the underlying asset for the pre-specified price, the exercised price, if the value of the asset drops below some level, the strike price. If the asset never falls below the strike price until maturity, the seller will earn the provision of selling this put option.

Thus, the bond investor accepts the risk that the revenue will drop below the debt service, and that risk is incorporated in the exercised price. The value of that option reflects the price of the default risk and it is determined as the average over all possible realizations. The realization is positive for the bondholder when he receives the payment as promised and there is no risk, thus the value of the option is 0. If the realization is negative, the bondholder will receive only available amount. Thus, the value of the option is equal to the difference between the strike price and the available amount.

In the context of the toll road and the revenue risk, the price of the option to accept the revenue risk is equal to:

$$\max \left(0, C_b(T_b - t_b) - \sum_{i=t_b}^{T_b} R_i \right) \quad (6-3)$$

where $C_b(T_b - t_b)$ is the remaining value of the debt repayment at time of default t_b and

$\sum_{i=t_b}^{T_b} R_i$ is the sum of the remaining revenue. The value of the risky bond P_r is then equal to

the value of a risk free bond P_f reduced for a value of the put option:

$$P_r = P_f - \max\left(0, C_b(T_b - t_b) - \sum_{i=t_b}^{T_b} R_i\right) \quad (6-4)$$

Following Eq. (6-1), the rate of return for the holder of the risky bond can be derived. Thus, the difference between the risky rate r_r and the risk free rate r_f is the credit spread. As discussed earlier, the focus in this research is on the pricing of the revenue risk and how the price of this risk is affected by the changes in the network topology. Hence, the credit spread is chosen as the measure of interest for bonds. The changes in the credit spread as the function of changes in the capacity on links is presented in the following section on the network example.

SENSITIVITY ANALYSIS

The set of input values for Monte Carlo simulation is presented in Table 6-1. Same as in the simulation for the sensitivity analysis for bank loans, the length of the debt service T_b is set to 10 so the later comparison of results for all considered financial instruments will have the same base case. For the same reason, other values are the same as they were used for bank loans. The additional input needed for the assessment and

testing of the credit spread is the risk free rate r_f . This value is also assumed, although the identification of the risk free rate of return for toll roads is subject of discussion, which is not addressed in this thesis. This value is taken to be the same as the drift rate for the GBM simulating the risk-free environment. The total value of debt scheduled for the repayment over time T_b is 16.8×10^6 .

Table 6-1. Summary of input parameters for Monte Carlo simulation for bonds

T_b	$R(0)$	μ	σ	T_r	$\frac{R(0)}{C_b}$	r_f	n
10	2.35×10^6	0.05	0.2	1	1.4	0.05	100,000

From Monte Carlo simulation, the credit spread is determined for the base case, that is, for the initial network topology. The credit spread for the given debt structure is 19 basis points (basis point is a measure commonly used for the credit spread and represents 1/100 of percent). The stress analysis is conducted for the same marginal increments as it was used in bank loans. The capacity values are varied from $C_{\min}=1,500$ to $C_{\max}=3,500$ with the marginal increment of $\Delta C=50$ on links 1 and 3, that is, the feeder and the competing link, respectively.

As mentioned earlier, it is expected that the increase in the capacity of the feeder link will positively affect risk parameters, thus add to the value of the project. As the capacity increase, the travel cost on the link decreases, thus attracting more traffic to it. As the number of trips on the feeder link increases, it will also increase the traffic on the

toll link and the revenue. The higher the revenue collected on the toll road, the risk of defaulting on the debt service will decrease. Thus, the credit spread should decrease. On the contrary, the increase in the capacity on the competing link will increase the credit spread.

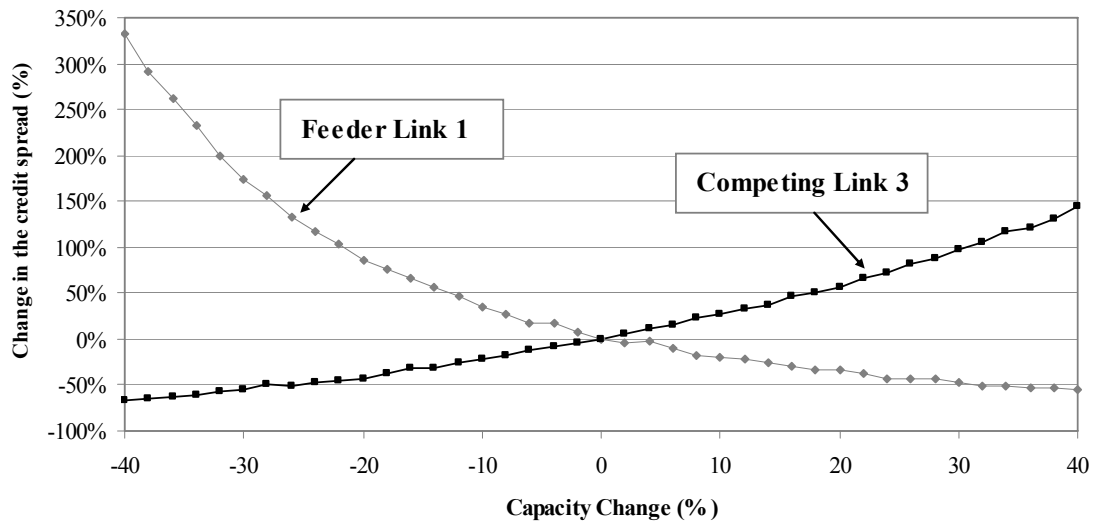


Fig. 6-2. Changes in the credit spread

Fig. 6-2 represents results of sensitivity analysis for the credit spread. The horizontal axis represents the percent change in the link's capacity, and the vertical axis represents the percent change in the credit spread. Results are as expected since the increase of the capacity on the feeder link decreases the price of the revenue risk, thus adding the value to the project. Increase in the capacity on the competing link increases the price of the risk, thus increasing the credit spread.

However, it should be noted, as with the bank loans, that changes in the credit spread with the respect to changes on the feeder and the competing link are asymmetric and non-linear. For example, the decrease in the capacity of the feeder link has higher marginal impact on the credit spread than increase in the capacity on the same link. Also, the network improvements, which are increasing the credit spread thus detracting from the project's value, on the feeder link have higher impact than improvements on the competing link. In other words, comparing with the base case, increase in the capacity on the competing link has a lower effect than decrease in the capacity on the feeder link. Results of the sensitivity analysis are presented in Table 6-2 for the better understanding of the magnitude of the effect.

Table 6-2. Summary of results for bonds sensitivity analysis

Capacity Change		Δ Credit Spread
Feeder Link $\Delta C1$	+10%	-20%
	-10%	+36%
	+20%	-34%
	-20%	+87%
Competing Link $\Delta C3$	+10%	+28%
	-10%	-21%
	+20%	+57%
	-20%	-42%

Decrease in the credit spread happens when the capacity is added to the feeder link or when the capacity of the competing link is reduced. For these cases, the

magnitude of the effect on the credit spread is similar for both network improvement actions. For example, change of the capacity by $\pm 10\%$, either on the competing or the feeder link, reduces the credit spread 20%. In contrast, the network improvement actions that increase the credit spread are different for those changes that increase the capacity on the feeder link and those that decrease the capacity on the competing link. Increasing the capacity on the competing link by 10% will increase the credit spread by 28%, while the same marginal change in the favor of decreasing the feeder link's capacity will increase the credit spread by 36%. This difference between impacts on the credit spread because of changes on the feeder and the competing links, the impact that reduces the value of the project, is further increasing with the higher percentile changes of capacities.

SUMMARY

Bonds, as the financial instrument for debt raising in PPP agreements, are analyzed in this chapter in the context of toll roads. The credit spread is identified as the difference between yields on the risk-free bond and on the risky bond. The change of the credit spread with the respect to changes on the feeder link and the competing link is assessed. This assessment and sensitivity analysis provided an insight into the phenomenon of impact that different network topologies have on the price of revenue risk thus adding to or subtracting from the value of the project. Next chapter continues the analysis of financial instruments used in PPP agreements by proposing the new

approach for the risk mitigation between the public sector and the private sector in the form of using the real options.

CHAPTER VII

SALVAGE AND BUY-BACK OPTIONS

Real options are used in the project finance as the tool for the risk mitigation. From the literature review, it has been noted that the form of the option used for the toll road agreements vary from different government guarantees to revenue sharing agreements. Also, different mathematical approaches are used. This led to the conclusion that the private sector and the public sector do not have the specific type or the model for the toll road agreements and that they may vary from the project to the project. Hence, the research in this area of financial instruments was focused on the development of the new approach for the option value assessment. Two options were analyzed: the option to abandon the project for the salvage value and the option to buy-back the project, as it is explain in detail in following section.

OPTIONS VALUATIONS

The option to abandon the project is an option that provides a right to the private sector to sell the project back to the public sector if the project does not generate enough profit. For the public sector, the option to abandon the project for salvage value is an obligation to pay an arranged price if the investor decides to exercise that option. From the private sector's perspective, having this clause in the agreement adds value to the project since it reduces the revenue risk.

For the public sector, the option to buyback the project is a right to acquire the project back to its ownership if the profit from its operation exceeds some predetermined level. In this case, the owner (the public sector) has a right to buy-back the project for some value and to continue to operate the project and collect all future revenues. The uncertain revenue process defines the value of the project also as an uncertain value. Since one of the parameters for the option valuation is the value of the underlying project, project's value is determined as the expected sum of future revenues.

Conditions under which options can be exercised are defined as an upper and a lower boundary. The average revenue (AR) over some pre-specified time horizon is compared with these boundaries. If the AR is below or above boundaries, options can be exercised. This approach, with the average revenue as one of conditions for options calculation, overcomes the problem of yearly traffic volatility, hence revenues volatility risk. It allows, over pre-specified period, for traffic flows and revenues to fluctuate and takes into the consideration only the average revenue value.

For considered period, AR is calculated using Monte Carlo simulation as the sum of the discrete values for yearly revenue for each simulated path and divided by the length of a time horizon. Once the value of the AR is evaluated and set, the next step is to, for those simulation paths for which forecasted revenue is above AR, compare the expected project value with the exercised price.

First, expected value of the integral of the GBM process is explained. This formulation is used for the calculation of the expected project's value over the remaining concession period. Consider a stochastic process in a risk neutral environment:

$$dS(t) = rS(t)dt + \sigma S(t)dW_t \quad (7-1)$$

where r is the risk-free rate and σ is the volatility. Let's define a time to the maturity of an option T with the assumption that r and σ are constant over time horizon $[t_0, T]$.

Consider the integral of a stochastic process over observed time horizon:

$$A(T) = \int_{t_0}^T S(y)dy \quad (7-2)$$

With the definition of the expected value of the underlying process:

$$E[S(T)] = S_{t_0} * \exp\left[\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right] \quad (7-3)$$

Eq. (7-2) becomes:

$$E[A(T)] = E\left[S_{t_0} \int_0^{T-t_0} \exp\left[\left(r - \frac{1}{2}\sigma^2\right)y + \sigma W_y\right] dy\right] \quad (7-4)$$

Geman and Yor (1993) showed that the first order moment of the process

$A_t^{(\nu)} = \int_0^t \exp[2(\nu s + W_s)] ds$ is equal to:

$$E(A_t^{(\nu)}) = \frac{1}{2(\nu+1)} [\exp(2t(\nu+1)) - 1] \quad (7-5)$$

where

$$\nu = \frac{2r}{\sigma^2} - 1 \quad (7-6)$$

Due to the scaling property of Brownian motion $W(t) = (1/a)W(ta^2)$ and using the change of variable $y = 4\nu/\sigma^2$ (Geman and Yor, 1993), Eq. (7-4) becomes:

$$E[A(T)] = \frac{4S_{t_0}}{\sigma^2} E \left[\int_0^{\frac{4}{\sigma^2}(T-t_0)} \exp(2(\nu y + W_y)) dy \right] \quad (7-7)$$

Consider $S_{t_0} = R_{t_0}$ and $A(T)$ to be value of the project, then the expected value of the project $E[PV]$ determined in the time t_0 is:

$$E[PV(t_0)] = \frac{4R(t_0)}{\sigma^2} \frac{[\exp(2h(\nu+1)) - 1]}{2(\nu+1)} \quad (7-8)$$

where

$$h = \frac{\sigma^2}{4}(T - t_0) \quad (7-8a)$$

$$\nu = \frac{2r}{\sigma^2} - 1 \quad (7-8b)$$

Time for which the average revenue value is calculated is defined as $[0, t_0]$ and an upper bound as UB and lower bound as LB. Both options are priced as European barrier options, which mean that they can be exercised only at the end of the option's life. Option to abandon the project is considered as down-and-in barrier option and option to buyback the project is up-and-in barrier option. Down-and-in barrier option is the option for which, if the average revenue drops below lower barrier (down), the option becomes alive (in). Similarly, the up-and-in barrier option is the option for which, if the average revenue exceeds upper barrier (up), the option becomes alive (in). Both options can be exercised at time t_0 . For each simulated path, if the AR for $[0, t_0]$ is above UB or below LB, the value of options is calculated comparing the expected value of the project for the remaining period and the set exercised price (Eq. 7-9 and Eq. 7-10):

buy-back option (European call option)

$$C = \left[\max \left(E[PV(t)] - K_c, 0 \right) \mid \overline{AR}^n(\Delta t) > UB \right] \quad (7-9)$$

for option to abandon (European put option)

$$P = \left[\max \left(K_p - E[PV(t)], 0 \right) \mid \overline{AR}^n(\Delta t) < LB \right] \quad (7-10)$$

where K_c is the exercised price for the buyback option, C is the value of corresponding options, K_p is the exercised price for the salvage option and P is the value of that option. The exercised price can be considered as the cost of the initial investment in the project (Garvin and Cheah, 2004).

The paths in which the buyback option is in the money (expected project's value is higher than the exercised price), the public sector can exercise the option. In those cases, the public sector expects that the project will generate more profit than the required payment for this option or the exercised price. The paths in which the salvage option is in the money (expected project's value is lower than the exercised price), the private sector can exercise the option. In this case, the private sector expects that the project will not generate enough revenues. The option valuation model is presented in Fig. 7-1.

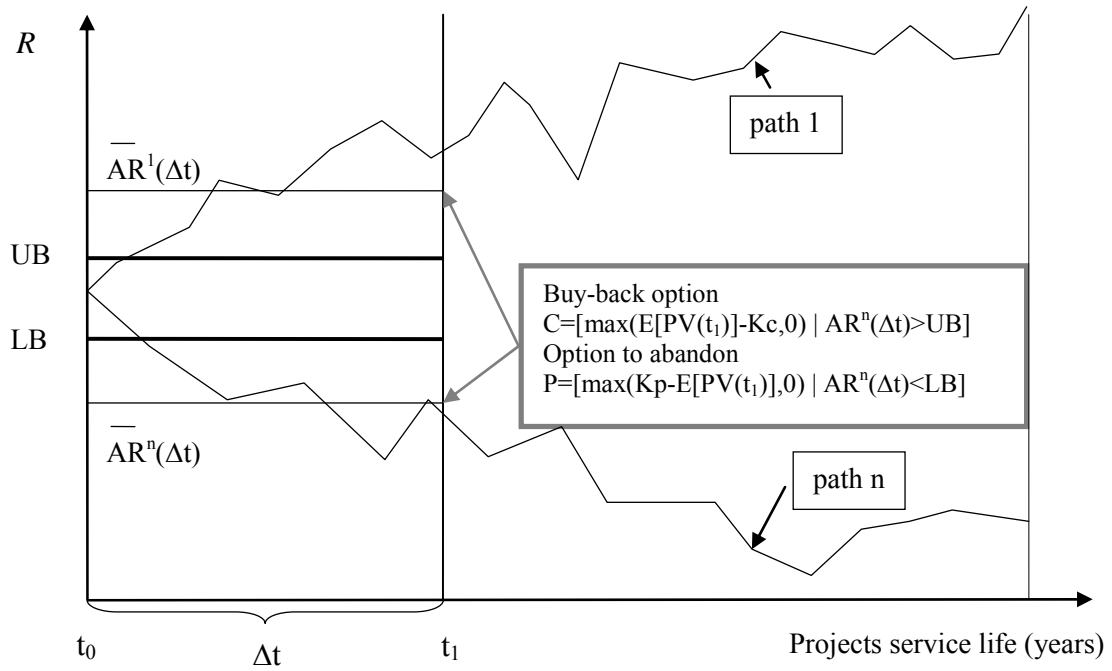


Fig. 7-1. Pricing the options

Further explanation of options pricing is addressed by observing one path of all simulated revenue paths, that is, path 1 on Fig. 7-1. After time Δt , when the option can be exercised, value of the $\overline{AR}^1(\Delta t)$ is determined for the period $[0, t_0]$ as an average of all revenues within that period. This average revenue is compared with the UB which is set in advance. Since the $\overline{AR}^1(\Delta t) > UB$, the option becomes alive. It can be exercised if it has positive payoff. For the simulated path 1, expected value of the project is determined from the Eq. (7-8) and compared with the exercised price K_c . Similarly, observing the path n, if the $\overline{AR}^n(\Delta t) < LB$ the salvage option will be exercised if the expected project's value is lower than the K_p .

SENSITIVITY ANALYSIS

The stress analysis is conducted to examine the effect of the change in the network structure on options values. The stress analysis is performed for capacity changes of one link in the network at the time. Similarly as in previous sensitivity analysis, link 1 and link 3 (Fig. 4-1) are used as the feeder link and the competing link, respectively. Capacities of these links are changed having all other values of network parameters being constant.

The set of input values for Monte Carlo simulation is presented in the Table 7-1. Values are the same as they were used for bank loans and bonds so the later comparison of results for all considered financial instruments will have the same base case. The additional input needed for the assessment and testing of the options values is the upper bound UB, the lower bound LB, the average revenue AR and exercised prices Kc and Kp. The number of simulation is $n = 100,000$ and the standard error $SE = 6.2 \cdot 10^3$.

Table 7-1. Summary of input parameters for Monte Carlo simulation for options

$R(0)$	μ	σ	r_f	LB	UB	Kc	Kp
$2.35 \cdot 10^6$	0.05	0.2	0.05	$3 \cdot 10^6$	$5 \cdot 10^6$	$20 \cdot 10^6$	$10 \cdot 10^6$

From Monte Carlo simulation, option values are determined for the base case, that is, for the initial network topology. The stress analysis is conducted for the same marginal increments as it was used in the analysis of bank loans and bonds. The capacity

values are varied from $C_{\min}=1,500$ to $C_{\max}=3,500$ with the marginal increment of $\Delta C=50$ on links 1 and 3, that is, the feeder and the competing link, respectively.

It is expected that any increase in the capacity on the feeder link will increase the traffic flow and the revenue on the toll link. Consequently, the expected value of the project will increase, which will reduce the value of the option to abandon the project and increase the value of the option to buy-back the project. In contrast, if the capacity of the feeder link decreases, it will reduce the traffic flow and the revenue on the toll link thus decreasing the project's expected value. This will cause the price of the option to abandon the project to alter and the price of the option to buy-back the project to decline.

For changes in the capacity on the competing link, it is expected that any increase in the capacity will reduce the expected project's value thus decreasing the value of the buy-back option and increasing the value of the salvage option. For network improvements which will lead to the decrease in the capacity on the competing link, it is expected that this improvements will increase the project's value. The price of the option to buy-back the project will increase and the price of the option to abandon the project will decrease. Results of the sensitivity analysis are presented in Fig. 7-2 for the option to buy-back the project. The horizontal axis represents the percent change in the link's capacity and the vertical axis is the percent of change in the option's value.

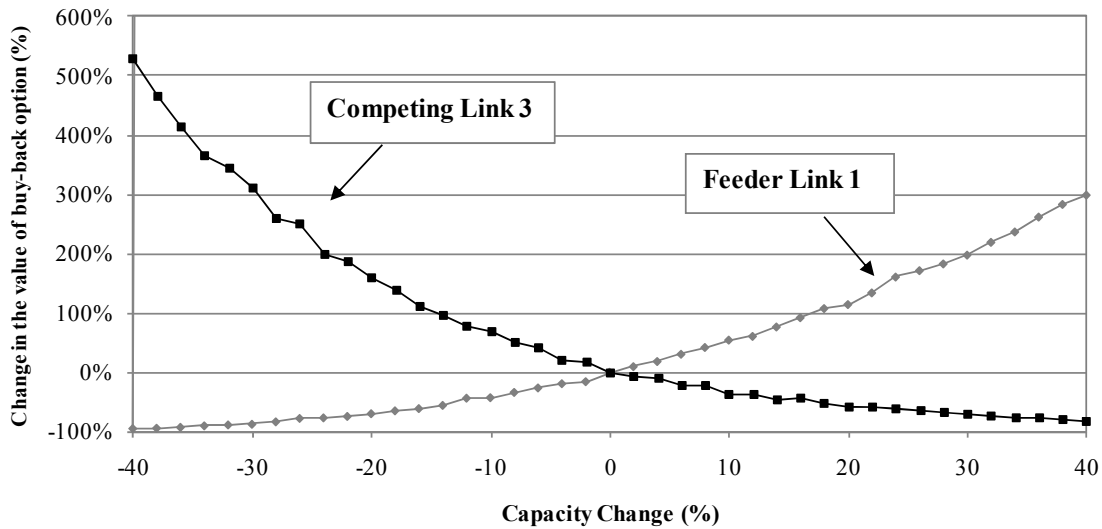


Fig. 7-2. Effect of the change in the capacity on the value of the option to buy-back

Results presented in Fig. 7-2 are as expected. The increase in the capacity on the feeder link caused the revenue of the project to increase, thus increasing the expected project's value. As the value of the project has been increasing, the price of the option to buy-back the project has also been increasing. In this case, the public sector has to pay the higher price to buy-back the project since it is expected that the project will generate more revenues and thus will be worth more. Although these changes in the network structure reduce the revenue risk, the price of this option is increasing because these network improvements add to the project's value and, since the price of the option is derived from the expected project's value, having this option in the agreement will be worth more. The same effect is achieved if the network structure is changed through the reduction in the capacity of the competing link. The revenue is increasing and the

project's value is increasing. The price of the option to buy-back the project which value is increasing will increase.

In contrast, the value of the project is decreasing if the capacity of the feeder link is decreasing or if the capacity of the competing link is increasing. These changes in the network structure will cause re-distribution of traffic thus reducing the number of trips on the toll link. The project's revenue will decrease as well as the expected project's value. The price of the option to buy-back the project will decrease since the project's worthiness is decreasing.

However, the effects of the network improvements on the value of the option to buy-back the project do not have the same magnitude nor are they symmetrical. For example, network improvements, which lead to the decrease in the capacity on the competing link, have higher impact on the value of the option than the increase in the capacity on feeder link. As the percent of change in the capacity on the competing link is altered, the effect on the price of the option is also altered. However, the same percent change on the feeder link will not have the same magnitude of the impact on the price of the option. Thus, for the option to buy the project, the decrease in the competing link has the higher impact on the option's value than increase in the feeder link.

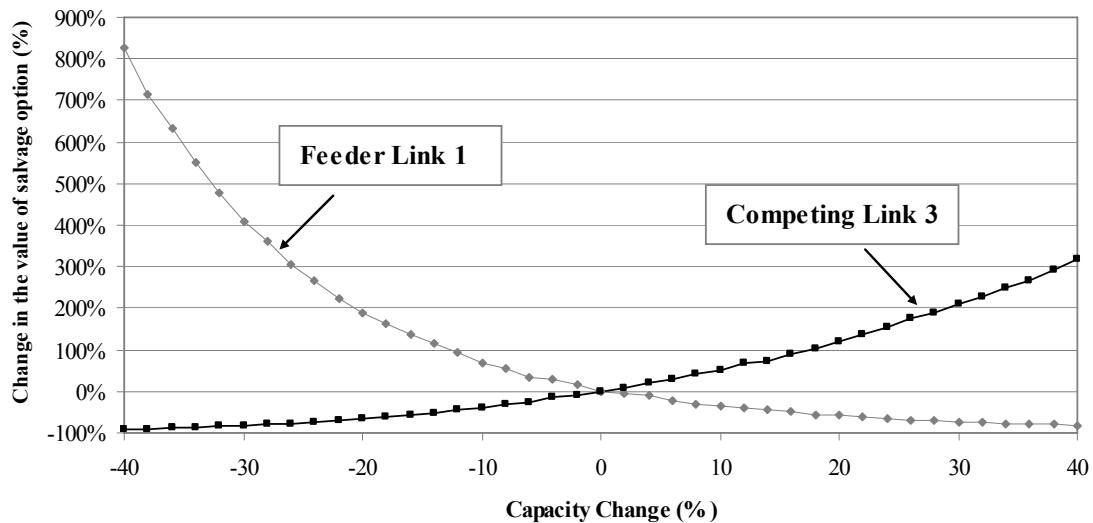


Fig. 7-3. Effect of the change in the capacity on the value of the salvage option

Fig. 7-3 represents the results of changes in the network structure on the value of the option to abandon the project for the salvage value. The horizontal axis represents the percent change in the link's capacity and the vertical axis is the percent of change in the option's value. Similarly, as in previous analysis, results are as expected. Increasing the capacity on the competing link or decreasing the capacity on the feeder link will cause the reduction of the revenue and the project's value. The price of the option to abandon the project which value is decreasing will increase. In contrast, if the value of the project is increasing by adding the capacity on the feeder link or reducing the capacity on the competing link, the project will worth more and thus the price of the option will decrease.

However, the results for the changes in the capacity on the competing and the feeder link are non-linear and asymmetrical. For example, marginal impact of the

reduced capacity on the feeder link is higher than the impact of the increase of the capacity on the competing link. Although both network improvements actions have the same effect, reducing the revenue on the toll road and thus reducing the project's value, the magnitude of the change on the option's price is not the same. For the option to abandon the project for the salvage value, the change on the feeder link has a higher effect.

For better understanding of the magnitude of the change in options values, results of the sensitivity analysis for both options are presented in Table 7-2.

Table 7-2. Summary of results for options sensitivity analysis

Capacity Change		Δ Value of the buy-back option	Δ Value of the salvage option
Feeder Link $\Delta C1$	+10%	+54%	-34%
	-10%	-43%	+69%
	+20%	+114%	-58%
	-20%	-70%	+190%
Competing Link $\Delta C3$	+10%	-38%	+49%
	-10%	+67%	-40%
	+20%	-58%	+118%
	-20%	+158%	-66%

For the value of the buy back option, decrease occurs when the capacity on the feeder link is decreased or when the capacity on the competing link is increased. For example, if the capacity on link 1 is increased by 10%, the value of the option will increase by 54%. The opposite marginal change on link 3 will increase by 67%. If the

magnitude of these changes rises to 20% than the value of the option will increase by 114%, for the change on link 1, and 158%, for the change on link 3.

In contrast, the increase in the value of the salvage option appears when the capacity increases on the competing link or decreases on the feeder link. Increase in the capacity of the competing link 3 by 10% will increase the value by 49%. If the capacity on the feeder link 1 is decreased by 10%, it will increase the value by 69%. Similarly, if these marginal changes on capacities rise to 20%, the impact on the value significantly alters. The increase in the value for the change on the link 3 is 118% while for the change on the link 1 is 190%. The impact is thus significantly different for network improvements actions on the feeder link 1 and the competing link 3.

SUMMARY

In this chapter, two options are presented as the tool used for the risk mitigation in the negotiation phase between the public sector and the private sector for toll road agreements. Both options are priced as the European boundary options. The approach for the option pricing with the use of the stochastic integral for GBM as the method for the calculation of the expected project's value is proposed. Sensitivity analysis is conducted for different network topologies and results are analyzed with the respect to changes in options values. This chapter concludes the analysis of three financial instruments used in toll road agreements: bank loans, bonds, and real options. The following chapter provides an overview and discussion of results.

CHAPTER VIII

OVERVIEW OF RESULTS

Sensitivity analyses are performed for marginal changes in the capacity on the feeder and the competing link identified in the simple network example. Each financial instrument is analyzed separately and the results were as expected. This has validated assumptions and mathematical models used to establish the base case as the case when there are no changes in the network.

Fig. 8-1 represents the overview of results obtained from the sensitivity analyses. The horizontal axis represents the percent of change in the capacity on the link and the vertical axis represents the percent change in the risk parameter. Risk parameters were identified in previous chapters for each financial instrument. For example, for bank loans, risk weighted assets (RWA) is used since it represents the risk weight of the loan in the bank's required capital. Credit spread is used as the risk parameter for bonds since it represents the difference on the rate of return between a risk free bond and a risky bond. Price of the buy-back and the salvage option is used as the risk parameter since the private sector and the public sector are interested in this value, as it is the price of the risk mitigation strategy.

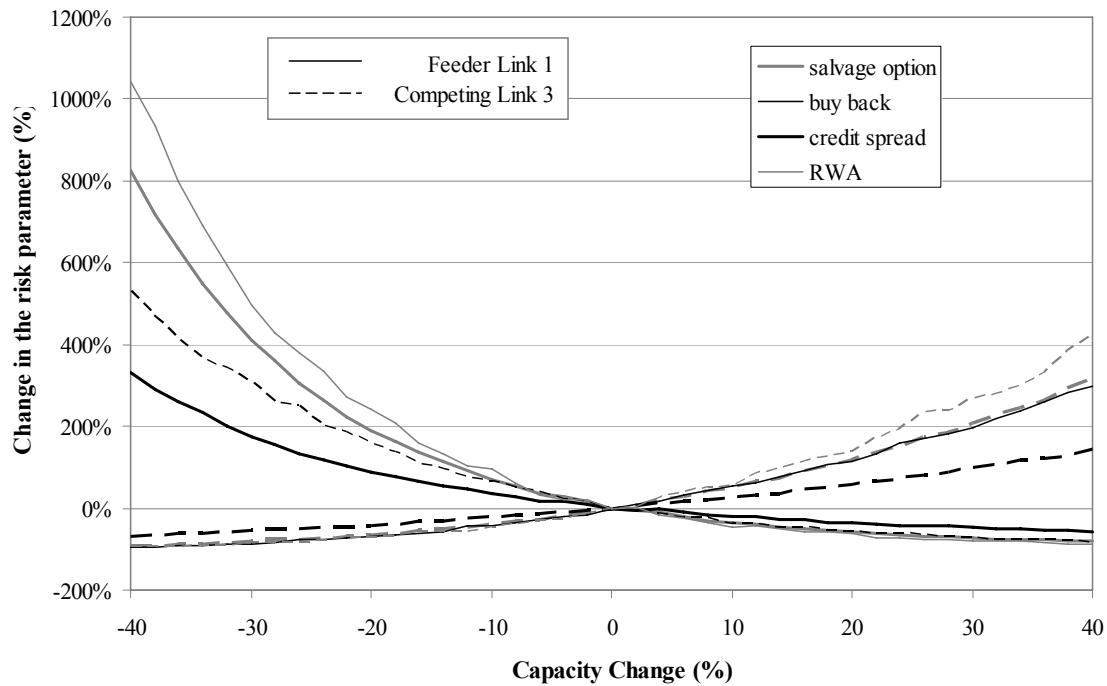


Fig. 8-1. Overview of results

Results indicate that the credit spread, as one of the analyzed risk parameters, is the least sensitive to the changes in the network structure. Changes in the network topology that increase the price of the revenue risk have a higher impact on the credit spread than changes that will decrease the credit spread. The same result is noticeable for other risk parameters: the effect of the increase in the price of the revenue risk is higher than the effect of the decrease in the price for the same marginal changes in the network structure.

Prices of options to buy-back the project or to abandon the project are next in the sensitivity categorization. Prices of these two options will be significantly altered by any change in the network that will increase the price. Depending on the type of the option

analyzed, the change in the feeder link or the change in the competing link will have a higher impact. For example, the price of the option to buy-back the project will be more altered by the decrease in the capacity on the competing link than the increase in the capacity on the feeder link. In contrast, the price of the salvage option will be more altered by the decrease in the capacity on the feeder link than the increase in the capacity on the competing link.

The risk parameter identified in the analysis of bank loans, RWA, is the most sensitive to the changes in the network structure. The change in the risk weight is significantly altered by the decrease in the capacity on the feeder link and the increase in the capacity on the competing link. However, changes on the feeder link that add to the price of the revenue risk have a higher impact than changes in the competing link with the same effect.

In the context of these results, project participants gain an information about the sensitivity of financial instruments to changes in the network topology. For example, if the private sector has a bank loan raised for the project, obligation to repay the project's bonds and a clause in the agreement with the public sector about the option to abandon the project, it should expect that the price of the bank's loan will be the most affected by the changes in the initial network topology. Also, changes on feeder links can be more important for consideration than changes on competing links, thus making the non-competing clause in the agreement, as defined in the literature review, insufficient and incomplete.

CHAPTER IX

CONCLUSIONS

In this chapter, the summary of this research is presented along with conclusions and recommendations for future research.

SUMMARY

Development, delivery, and operation of public infrastructure is becoming increasingly dependent on participation of the private sector. While revenue generating projects such as toll roads were traditionally developed and funded from the public sources, in recent years, as the public demand for new projects have exceeded the ability of the public sector to deliver them, the private investors have started to fulfill a gap between the needed and the available infrastructure.

However, the participation of the private sector in PPP agreements is constrained by the availability of funds thus putting the financial institutions in the role of lenders in these agreements. The scheme of the PPP agreements and the interaction among project's participants are very complex. Due to this complexity, there are numerous associated risks, but one risk can be distinguished as the core of the project's financial feasibility: the revenue risk.

This research examines the correlation of the revenue risk and the three most commonly used financial instruments in PPP agreements: bank loans, bonds and real

options. The research problem was identified as the need for the identification and the quantification of the effect that changes in the network structure has on the project's financial feasibility. All project participants are affected by changes in the network topology, but the remaining question is: what is the impact of these changes on project participants? It is considered that all participants are most interest in the financial aspect of the project and financial instruments used in PPP agreements. Hence, the research question is formulated as: *how will these network improvements affect the revenue on the toll road and the project's financial feasibility?* The objective of the research was to develop the method that will allow an assessment of this effect on above-mentioned financial instruments.

For better understanding of the effects that different network topologies have on project's financial viability, the sensitivity analysis is conducted for different marginal changes in the capacities on the feeder and the competing links. The overview of results is presented in the following section.

CONCLUSIONS

The objective of this research was to develop the method that will allow an assessment of the effect that the public sector decisions regarding the network improvements have on the toll road financial feasibility. Financial feasibility is defined through the identification of the three most commonly used financial instruments in PPP agreements. The proposed method was tested through the sensitivity analysis with

respect to different network topologies. Obtained results were as expected which was taken as the validity test of the method. Thus, it is anticipated that the proposed method is a valuable tool, which will add to the current practice because it has the following properties:

- It creates a solid basis for the objective assessment of the impact that decisions made by the public sector, regarding the network improvements, have on financial instruments;
- It provides a tool for quantification of the impact that these changes in the network structure have on the project's financial feasibility; and
- It builds a common ground which ensures that the decisions made by the public sector are interpreted in a meaningful manner for project managers and decision makers, that is, the public sector, the private sector and financial institutions.

Obtained results from the sensitivity analysis indicate that the changes in the network structure have an impact on the projects financial feasibility. However, the magnitude of this impact depends on the type of the network improvement (decreasing or increasing the capacity on the link), the position of the link in the network (the feeder or the competing link) and the type of financial instrument. From obtained results and provided analyses, following conclusions are derived:

- Consideration and identification of feeder and competing links in the network is an important topic which should be considered in the negotiation phase between all project participants;

- Among analyzed financial instruments, the credit spread is the least sensitive to the network improvements and the risk weighted asset is the most sensitive;
- Increasing the revenue risk price is easier to achieve by implementing the network improvement actions than to decrease the price.

RECOMMENDATIONS FOR FUTURE RESEARCH

This research has been concluded with the development of the method for the assessment of the effect that different network improvement actions have on selected financial instruments. The analysis of the financial instruments, in future research, could be expanded to include sensitivity of the rate of return on the equity invested. Also, the analysis of the above mentioned effects on the operation and maintenance costs can provide valuable information for project participants.

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